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A Mathematical Formulation for Planning Automated Aircraft Separations for AERA 3

Advanced System Design Service
Federal Aviation Administration
Washington, D.C. 20591

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16. Abstract

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Though other algorithms have been proposed for this purpose, GS uniquely facilitates quantitative analysis of the conflict resolution process itself, as well as quantitative analysis of the links between pairwise conflict resolution and longer lookahead air traffic control (ATC) strategies.

The main result of this document is a closed-form mathematical formula which relates parameterizations of (a) the encounter geometry, (b) the pathkeeping uncertainties, (c) the minimum separation achieved by GS, and (d) the "gentleness" of parallel lateral offset resolution maneuvers (as parameterized by magnitude of offset and by induced delay upon the aircraft).

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ABSTRACT

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Though other algorithms have been proposed for this purpose, GS uniquely facilitates quantitative analysis of the conflict resolution process itself, as well as quantitative analysis of the links between pairwise conflict resolution and longer lookahead air traffic control (ATC) strategies.

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REPRESENTATIVE NUMERICAL RESULTS

The following are typical numerical examples of GS results. Consider any conflict between two aircraft at en route speeds (say, 400 knots or above), whose pathkeeping uncertainties are ± 0.5 nmi laterally and ± 1.0 nmi longitudinally. It is assumed that the aircraft paths are linear and no third aircraft are in the vicinity of the conflict. Then,

(i) GS can always achieve 5 nmi of net horizontal separation by maneuvering one aircraft no more than 12 nmi left or right of its nominal path, suffering an induced delay of no worse than 20 seconds, as long as either:

- The encounter angle is 45 degrees or more, or
- The ratio of the slower aircraft's speed to that of the faster is less than 0.84.

(ii) In any crossing geometry, GS can achieve 5 nmi of net horizontal separation by maneuvering one aircraft no more than 12 nmi left or right of its nominal path, suffering an induced delay of no worse than (approximately) 50 seconds.

Results such as (i) and (ii) are calculable offline. In particular, they are applicable to conflicts well in the future (e.g., over MOM's 30-minute horizon), even though the maneuver need start only a few minutes prior to the conflict.

GS-MOM INTERACTION

A symbiosis is established between GS and MOM. GS relies on MOM to simplify multiaircraft problems, so that only isolated, pairwise conflicts remain to be resolved at GS's timeframe (several minutes lookahead). In return, GS resolution maneuvers have a quantifiably limited downstream trajectory impact (i.e., they are gentle, in a precisely-defined sense), which improves the integrity of the trajectory data used by long-lookahead functions such as MOM to do their jobs.

GS PERFORMANCE AS A LOWER BOUND ON AERA 3 PERFORMANCE

Although AERA 3 need not always use the particular resolution maneuver generated by GS for an aircraft-to-aircraft conflict, it would fail to do so only if it finds something better (e.g., some different lateral resolution or some change in vertical rate or speed, judged to be more effective in some sense than GS's resolution). Therefore, the GS result serves as a lower bound not only for GS performance but also for AERA 3 resolution of pairwise linear-path conflicts as a whole.

EXECUTIVE SUMMARY

INTRODUCTION

An algorithm, called Gentle-Strict (GS), is given for automated resolution of crossing conflicts between two aircraft, using parallel lateral offset maneuvers.

Though other algorithms have been proposed for this purpose, GS uniquely facilitates quantitative analysis of the conflict resolution process itself, as well as quantitative analysis of the links between pairwise conflict resolution and longer-lookahead air traffic control (ATC) strategies.

MAIN RESULTS

The main result of this document is a closed-form mathematical formula which relates parameterizations of (a) the encounter geometry, (b) the pathkeeping uncertainties, (c) the minimum separation achieved by GS, and (d) the "gentleness" of parallel lateral offset resolution maneuvers (as parameterized by magnitude of offset and by induced delay upon the aircraft).

Given pathkeeping uncertainties (b) and a desired minimum separation (c). for instance, one can determine either:

- Bounds on gentleness (d) as a function of encounter geometry (a), or
- The subset of the state space of encounter geometries (a) which is resolvable via parallel offset maneuvers meeting given bounds on gentleness (d).

BACKGROUND

The Federal Aviation Administration (FAA) is sponsoring research by MITRE into ATC automation to achieve goals such as:

- Improved services to public/pilots/airlines (i.e., fewer delays, avoidance of unnecessary maneuvers, ability to grant pilot-requested routes whenever possible).
- Increased airspace capacity and utilization.
- Increased safety.
- Increased productivity (more aircraft moved per person).

An important part of the ATC automation effort is Automated En Route ATC, or AERA, which will enhance the current ATC system in successive phases (AERA 1, 2, and 3). Each successive phase of ATC automation, through AERA 3, is expected to achieve the above goals to a greater degree.

Gentle-Strict is developed as a tool for AERA 3.

OUTLINE OF AERA 3

In AERA 3 the automation assumes, for the first time, responsibility for separating aircraft. Detection and resolution of aircraft-to-aircraft conflicts is completely automated. Humans participate in planning but (unlike earlier phases of AERA) are not involved in time-critical separation assurance decisions.

AERA 3 has a hierarchy of functions that can be compared to nested shells; each shell relies upon the next-outer shell to assure that it is not given a problem too difficult to handle.

The innermost shell, called Automated Separation Function (ASF), resolves pairwise separation problems, generally considering them one at a time. ASF looks a few minutes into the future.

The next shell out, Maneuver Option Manager (MOM), considers interrelated pairwise possible separation problems among sets of aircraft. MOM determines a set of maneuvers (called outs) whereby such interrelated possible problems are simplified into individual pairwise possible problems, thereby helping to justify from a systems planning point of view ASF's one-at-a-time approach. MOM looks about 30 minutes into the future.

The third (and outermost) shell of AERA 3 is known as the Airspace Manager Planning Functions, whose lookahead period is perhaps 90 minutes. This shell consists of automation aids to be used by a human (known as the Airspace Manager). This shell (among other things) prevents traffic from becoming too dense for MOM to handle (e.g., so that aircraft have too few outs).

MOTIVATION FOR GS WORK

GS has applications both in the ASF and MOM shells in AERA 3. For the ASF shell, GS provides solutions to specific conflicts that occur a few minutes in the future. For MOM, GS provides bounds on the magnitude of resolution maneuvers needed for possible conflicts up to about 30 minutes in the future. Also, in certain circumstances when MOM finds no outs for a given aircraft, GS often guarantees that an out will become available with the passage of time.

There is, in addition, a deeper motivation underlying the GS work. To help verify AERA 3, given AERA 3's significant increase in automation responsibility, it is important to establish a firm theoretical basis for conflict resolution (which, as practiced by human controllers today, and by algorithms based upon human expertise, is something of an art). An attractive approach may therefore be to consider an algorithm, such as GS, whose foundation rests firmly upon mathematics,

SECTION 1

INTRODUCTION

An algorithm, called Gentle-Strict (GS), is given for automated resolution of crossing conflicts between two aircraft.

Though other algorithms have been proposed for this purpose, GS uniquely facilitates quantitative analysis of the conflict resolution process itself, as well as quantitative analysis of the links between pairwise conflict resolution and longer-lookahead air traffic control (ATC) strategies.

GS relates parameterizations of the following items concerning an aircraft-to-aircraft crossing conflict via closed-form mathematical formulae:

(a) The encounter geometry (e.g., encounter angle, relative speed, relative timing to intersection; aircraft paths are assumed linear).

(b) The uncertainty in the aircraft predicted positions during the conflict ("strictness" by which the aircraft follows its plan).

(c) The separation achieved between the aircraft (due to a GS maneuver).

(d) The "gentleness" of the resolution maneuver (assumed to be a parallel lateral offset), as parameterized by:

- Magnitude of the parallel lateral offset
- Delay induced by the parallel offset

Given uncertainties (b) and a desired minimum separation (c), for instance, one can determine bounds on gentleness (d) as a function of encounter geometry (a). Or, given (b) and (c), one can determine what subset of the state space of encounter geometries (a) is resolvable via parallel offset maneuvers meeting a given bound on gentleness (d).

1.1 BACKGROUND

The Federal Aviation Administration (FAA) is sponsoring research by MITRE into ATC automation to achieve goals such as:

- Improved services to public/pilots/airlines (i.e., fewer delays, avoidance of unnecessary maneuvers, ability to grant pilot-requested routes whenever possible).

- Increased airspace capacity and utilization.

- Increased safety.
- Increased productivity (more aircraft moved per person).

An important part of the ATC automation effort is Automated En Route ATC, or AERA ([1], [2], [3]) which will enhance the current ATC system in successive phases (AERA 1, 2, and 3). Each successive phase of ATC automation, through AERA 3, is expected to achieve the above goals to a greater degree.

Gentle-Strict is developed as a tool for AERA 3.

1.1.1 Outline of Planned ATC Automation Prior to AERA 3

Today's ATC system relies principally upon human controller expertise, at all levels (national, local, and sector), with minimal assistance from automation. Safety is maintained, but often at the cost of reduced capacity and benefits to users. Many of the limitations of today's system, which stem from limitations on manual and mental processing capabilities of a human being, can be overcome by automation. To help accomplish such automation, the FAA has developed the National Airspace System Plan [4], of which AERA is a part.

The first phase of AERA, AERA 1, introduces a number of new capabilities, most importantly a notice to controllers of upcoming conflicts between pairs of aircraft. The second phase of AERA, AERA 2, also introduces many new capabilities, notably a list of suggested resolution maneuvers for upcoming conflicts. Ultimate responsibility for aircraft separation remains with the human sector controller, under AERA 1 and 2 just as today.

1.1.2 Outline of AERA 3

In AERA 3 the automation assumes, for the first time, responsibility for separating aircraft. Detection and resolution of aircraft-to-aircraft conflicts is completely automated. Humans participate in planning but (unlike earlier phases of AERA) are not involved in real time-critical separation assurance decisions.

AERA 3 has a hierarchy of functions that can be compared to nested shells; each shell relies upon the next-outer shell to assure that it is not given a problem too difficult to handle.

The innermost shell, called Automated Separation Function (ASF), resolves pairwise separation problems, generally considering them one at a time. ASF looks a few minutes into the future.

The next shell out, Maneuver Option Manager (MOM) [5], considers interrelated pairwise possible separation problems among sets of aircraft. MOM determines a set of maneuvers (called outs) whereby such interrelated possible problems are simplified into individual pairwise possible problems, thereby helping to justify from a systems planning point of view ASF's one-at-a-time approach. MOM looks about 30 minutes into the future.

The third (and outermost) shell of AERA 3 is known as the Airspace Manager Planning Functions, whose lookahead period is perhaps 90 minutes. This shell consists of automation aids to be used by a human (known as the AREA Manager). This shell (among other things) prevents traffic from becoming too dense for MOM to handle (e.g., so that aircraft have too few outs).

The ASF and MOM shells are completely automated.

1.2 PURPOSE AND SCOPE OF DOCUMENT

This document presents the GS mathematical results, and shows how they can be useful for both the ASF and MOM shells in AERA 3. For the ASF shell, GS provides solutions to specific conflicts that occur a few minutes in the future. For MOM, GS provides bounds on the magnitude of resolution maneuvers needed for possible conflicts up to about 30 minutes in the future. Also, in certain circumstances when MOM finds no outs for a given aircraft, GS often guarantees that an out will become available with the passage of time.

Sections 2 through 5 present the significant GS concepts and give an overview of the main results. These sections are intended for a general audience having an interest in ATC automation. It is not assumed that the reader has detailed knowledge of the current ATC system. GS's contribution to AERA 3's ASF shell and to AERA 3's MOM shell are discussed.

The appendices present the detailed mathematical derivations and results. Certain subtopics relating to GS that arise in sections 2 through 5 are outlined there but discussed at greater length in the appendices. The appendices present possible extensions to GS work, include a glossary of terms and mathematical symbols, and are mainly intended for those who are interested in conflict resolution algorithms.

1.3 MOTIVATION FOR GS

Several motivations for GS have been mentioned above, such as assisting ASF to resolve certain conflicts, and assisting MOM in certain situations where no outs are available.

There is also a deeper motivation. To help verify AERA 3, given AERA 3's significant increase in automation responsibility, it is important to establish a firm theoretical basis for conflict resolution (which, as practiced by human controllers today, and by algorithms based upon human expertise, is something of an art). The capture of increasingly sophisticated levels of controller expertise in resolving conflicts (the paradigm in AERA 2, in which the automation recommends resolutions to the controller, who is still in the loop [2], [3]) may not be the best approach for AERA 3. First of all, the very fact of automating conflict resolution alters many of the constraints on the problem, some of which have been woven deeply into the controllers' knowledge. It is not clear if the appropriate portions of the controllers' knowledge, and those portions only, can be fully automated. Secondly, even if the controllers' knowledge were fully applicable to AERA 3, a fully automated expertise-based algorithm might be difficult to validate and verify.

A better approach may be to consider an algorithm whose foundation rests firmly upon mathematics, rather than upon human expertise. Such an algorithm may be inherently easier to analyze, explain, code, validate, verify, etc. Ideally, at least part of the verification process would be accomplished by mathematical proof, rather than by sole reliance on more standard methodologies (such an exhaustive testing). These and related ideas are discussed in more detail in Appendix A.

1.4 OVERVIEW OF MAIN RESULTS

The main result of this document is a closed-form mathematical formula which relates (a), (b), (c), and (d) above—that is, parameterizations of the encounter geometry, the pathkeeping uncertainties, the minimum separation achieved by a GS resolution, and the gentleness of parallel lateral offset resolution maneuvers (as parameterized by magnitude of offset and by induced delay upon the aircraft).

It is assumed that in the immediate vicinity of the conflict, the two aircraft fly linear trajectories, and no third aircraft is laterally nearby.

The following numerical examples of GS results give a good intuitive idea of what is to come. Aircraft pathkeeping uncertainties are assumed to be ± 0.5 nmi laterally and ± 1.0 nmi longitudinally. Then:

(i) Consider any conflict between aircraft at en route speeds (say 400 knots or above). GS can always achieve 5 nmi of net horizontal separation by maneuvering one aircraft no more than 12 nmi left or right of its nominal path, suffering an induced delay of no worse than 20 seconds, as long as either:

- The encounter angle is 45 degrees or more, or
- The ratio of the slower aircraft's speed to that of the faster is less than 0.84.

(ii) If the induced delay is expressed as an along-route distance (2.2 nmi) rather than a time interval (20 seconds), (i) is true as well for aircraft at speeds slower than 400 knots.

(iii) In any crossing geometry, GS can achieve 5 nmi of net horizontal separation by maneuvering one aircraft no more than 12 nmi left or right of its nominal path, suffering an induced delay of no worse than (approximately) 50 seconds.

Results such as (i), (ii), and (iii) are calculable offline. In particular, they are applicable to conflicts well in the future (e.g., over MOM's 30-minute lookahead horizon), even though the maneuver need start only a few minutes prior to the conflict.

A symbiosis is established between GS and MOM. GS relies on MOM to simplify multiaircraft problems, so that only isolated, pairwise conflicts remain to be resolved at GS's timeframe (several minutes lookahead). In return, GS resolution maneuvers have a quantifiably limited downstream

trajectory impact (i.e., they are gentle, in a precisely defined sense), which improves the integrity of the trajectory data used by long lookahead functions such as MOM to do their jobs. The symbiotic cycle is shown graphically in Figure 1-1.

Although AERA 3 need not always use the particular resolution maneuver generated by GS for an aircraft-to-aircraft conflict, it would fail to do so only if it finds something better (e.g., some different lateral resolution or some change in vertical rate or speed, judged to be more effective in some sense than GS's resolution). Therefore, the GS result serves as a lower bound not only for GS performance but also for AERA 3 resolution of pairwise linear-path conflicts as a whole.

1.5 ORGANIZATION OF DOCUMENT

Section 2 introduces the GS algorithm, and introduces the parameters to characterize an encounter geometry and to characterize gentleness. Section 3 presents the mathematical results (without proof; for this, see Appendix F). Section 4 discusses certain open issues and possible further work. Section 5 provides a summary.

Appendix A discusses why a mathematical-based algorithm (as opposed to one based upon human expertise) may have certain advantages for AERA 3. Appendix B presents methods by which GS might be extended to geometries involving route bends and nearby traffic. Appendix C presents an overview of Maneuver Option Manager (discussed in detail in [5]). Appendix D discusses the symbiosis between GS and Maneuver Option Manager. Appendix E presents further discussion involving the parameterization of gentleness. Appendix F presents the main mathematical derivations for the GS results, including the proofs for the results given in Section 3. Appendix G presents further discussion regarding the modeling of longitudinal and lateral uncertainties. Appendix H is a glossary of GS terms and key mathematical symbols. Appendix I lists references.

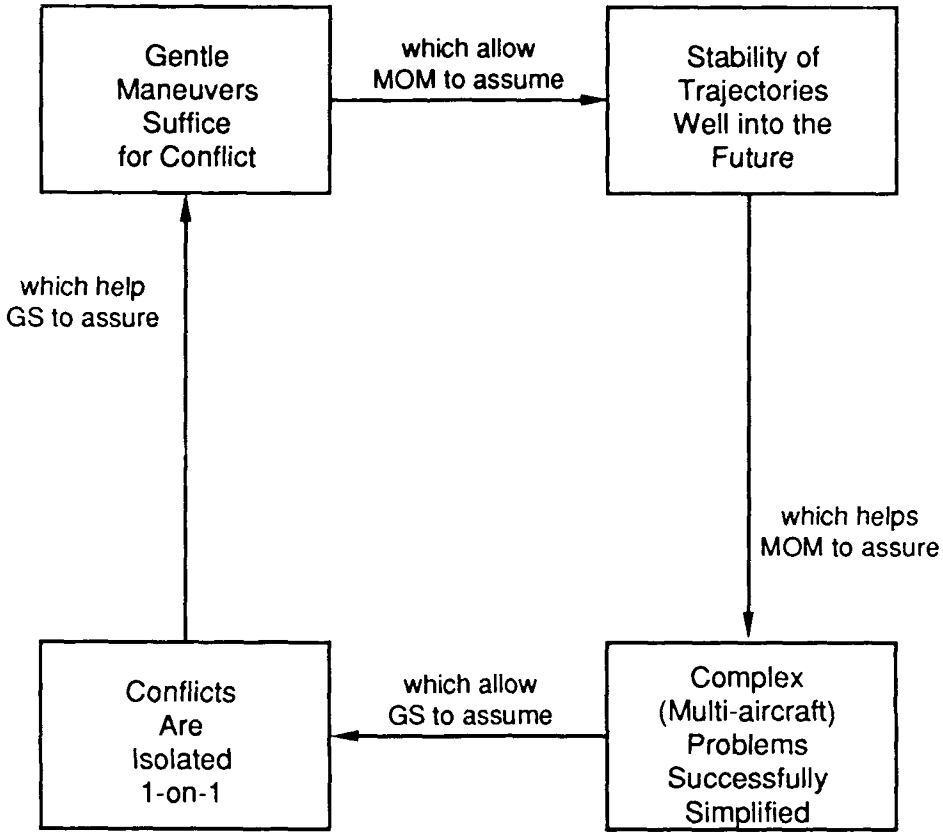

FIGURE 1-1
SYMBIOSIS BETWEEN MOM AND GS

SECTION 2

OVERVIEW OF GS ALGORITHM, PARAMETERS AND NOTATION

In this section, an outline of the GS algorithm is presented, as are GS's key parameters, terminology and notation.

2.1 OVERVIEW OF GS ALGORITHM

GS resolutions typically entail:

(A) a parallel lateral offset maneuver for one aircraft, which is "gentle" in a precisely defined sense, and
(B) a strictly monitored directive to the other aircraft to avoid deviations from nominal of more than a tolerance in the "unsafe" directions (e.g., "Avoid lateral deviations left of nominal and avoid increases in airspeed").

If possible, (B) is given to both aircraft, avoiding even a gentle maneuver as in (A).[1] Safe separation can often be achieved in this fashion without any maneuver, via prohibiting (for both aircraft) deviations from nominal in unsafe directions. Such directives are not used routinely by controllers today (it would often burden them with significant extra workload, especially for aircraft lacking highly accurate navigation), nor are such directives planned for FAA-sponsored ATC automation on a nearer term than AERA 3. Hence, it is worth noting that this maneuverless GS technique is in and of itself a gain in user benefits (without it one of the aircraft must maneuver). GS will take advantage not only of expected navigation improvements for most aircraft by 2000, but also of the computer's tireless capacity to monitor that the aircraft do not make the prohibited deviations (and to uplink any necessary corrections directly to the pilot). The mathematical conditions where such a maneuverless resolution suffices are discussed in more detail in Appendix F, Section 10.

The bulk of this document analyzes situations where GS must actively maneuver an aircraft (gently, via (A)) to achieve adequate separation, while adopting strict pathkeeping (B) for the other aircraft. The focus is therefore on more "pessimistic" conflict geometries, i.e., those which tend to be the most difficult to resolve gently.

The mathematical analysis also yields a means of calculating the gentlest possible maneuver needed in a given geometry, to provide a given level of guaranteed separation.

The high-level GS algorithm is illustrated in Figure 2-1. If the predicted (horizontal) miss distance is large, intermediate, or small (relative to thresholds), the GS recommendation would be (respectively) no action, (B) to both pilots, or (A) to one pilot and (B) to the other.

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[1]Symmetry suggests versions of (A) and (B) in the vertical dimension as well–an idea that may merit further research, but is not pursued here.

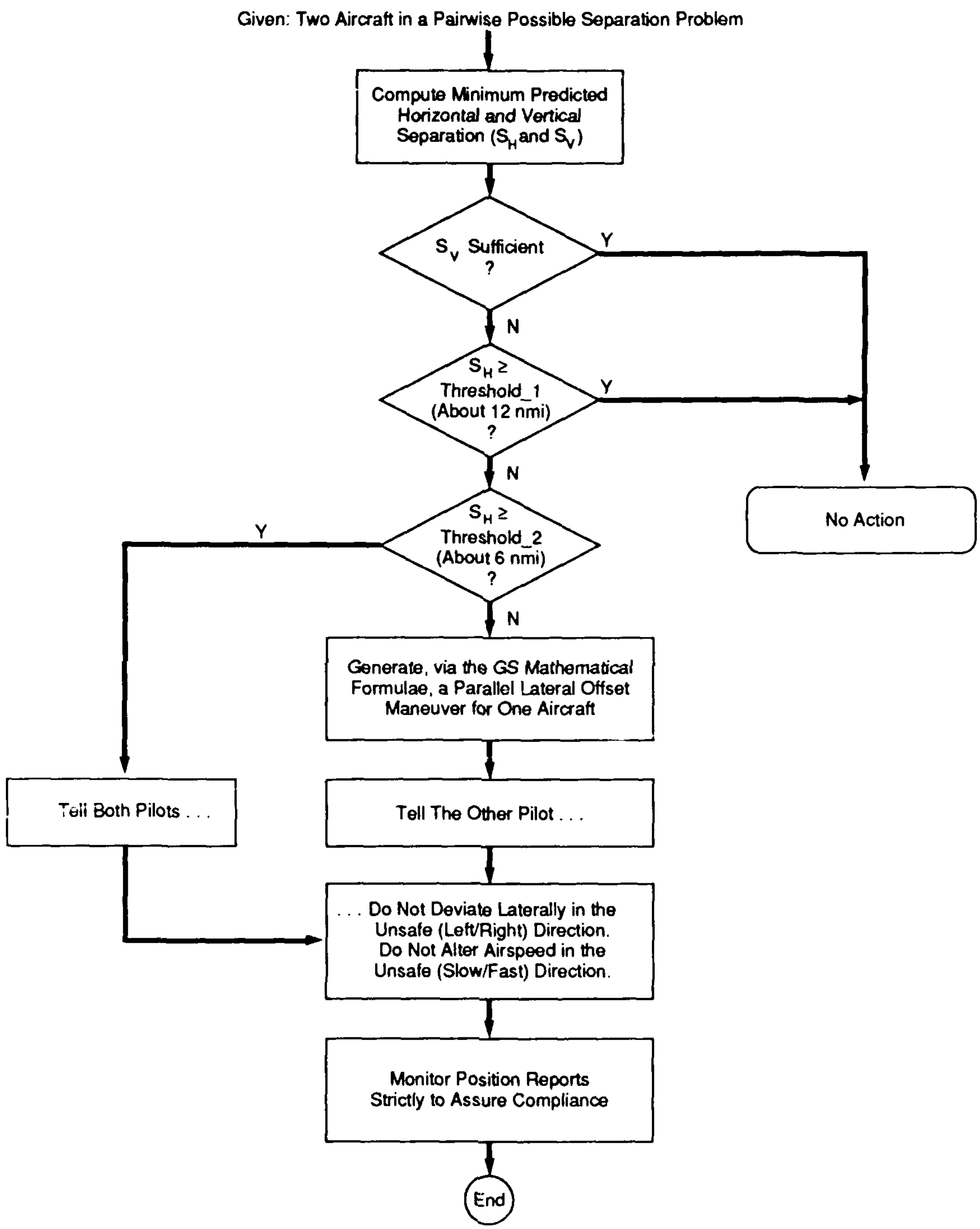

FIGURE 2-1
GS AS USED TO RESOLVE A PAIRWISE POSSIBLE SEPARATION PROBLEM

In the GS analysis, aircraft are assumed to follow their trajectories to within a specified tolerance. They are assumed to be free of interactions with third aircraft or other obstacles (e.g., third aircraft do not come within a parameter distance left or right of the two aircraft, in the vicinity of the conflict). The trajectories of the two aircraft are assumed to be free of horizontal accelerations (route bends, speed changes) in the vicinity of the conflict.

It is worth noting that gentle lateral maneuvers, the basis for the GS results documented here, are advantageous for other reasons, which apply to current as well as future ATC. First, gentle maneuvers (or directives not to maneuver) are favored by users over more significant maneuvers, which may result in increased fuel or time penalties. Generally speaking, the gentler the maneuver, the less the penalty. For en route aircraft, lateral maneuvers are often preferred by airlines over possible alternatives (speed and vertical maneuvers) due to fuel costs, unless an aircraft must change altitude or speed soon anyway (i.e., for reasons unrelated to the conflict, involving pilot preference or ATC necessity).

2.2 GS MANEUVERS

Figure 2-2 illustrates a GS resolution maneuver in parametric form. The original trajectory, HJK, is linear. The GS maneuver begins at H and reaches its full parallel offset at K_o (abreast of J). The magnitude of the offset (JK_o) is denoted u. The point K (on the original trajectory) is defined so that the path lengths HK and HK_o are equal. The maneuver, then, induces a delay in the aircraft's forward progress (northward progress, in the figure) of distance JK (denoted v). GS ignores the fact that the winds may affect the aircraft somewhat differently along HK versus HK_o, so that the induced delay may be slightly different from JK. However, this effect is minimized by the fact that u is typically much less than HJ in a GS maneuver. Also, the GS analysis includes explicit pathkeeping uncertainty terms, which can include an allowance for this effect.

There are many different ways for an aircraft to go from H to K_o, for any particular choice of u and v. GS makes no assumptions about what way might be used; the optimal path may vary with particular aircraft characteristics. GS does assume, however, that the point of minimum separation as a result of a GS maneuver occurs at or beyond K_o (the mathematical tractability hinges upon this assumption). The aircraft must therefore reach K_o at a time early enough for this assumption to be true (techniques are derived later to determine this time).

The aircraft is assumed to maintain (during and after the GS maneuver) its original speed and its original vertical rate, as a function of time. Figure 2-3 illustrates a pairwise conflict, for which both a leftward and a rightward GS maneuver are shown. The leftward maneuver causes the intersection to move (from P) to P_i; the rightward maneuver to P_o.

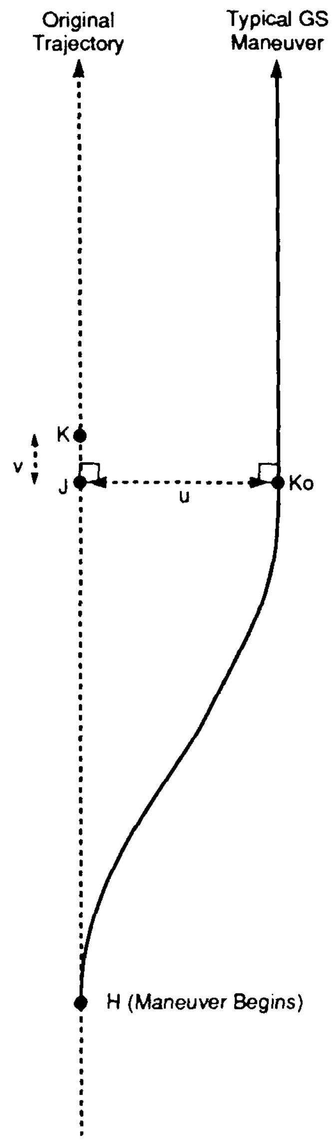

FIGURE 2-2
A TYPICAL GS MANEUVER

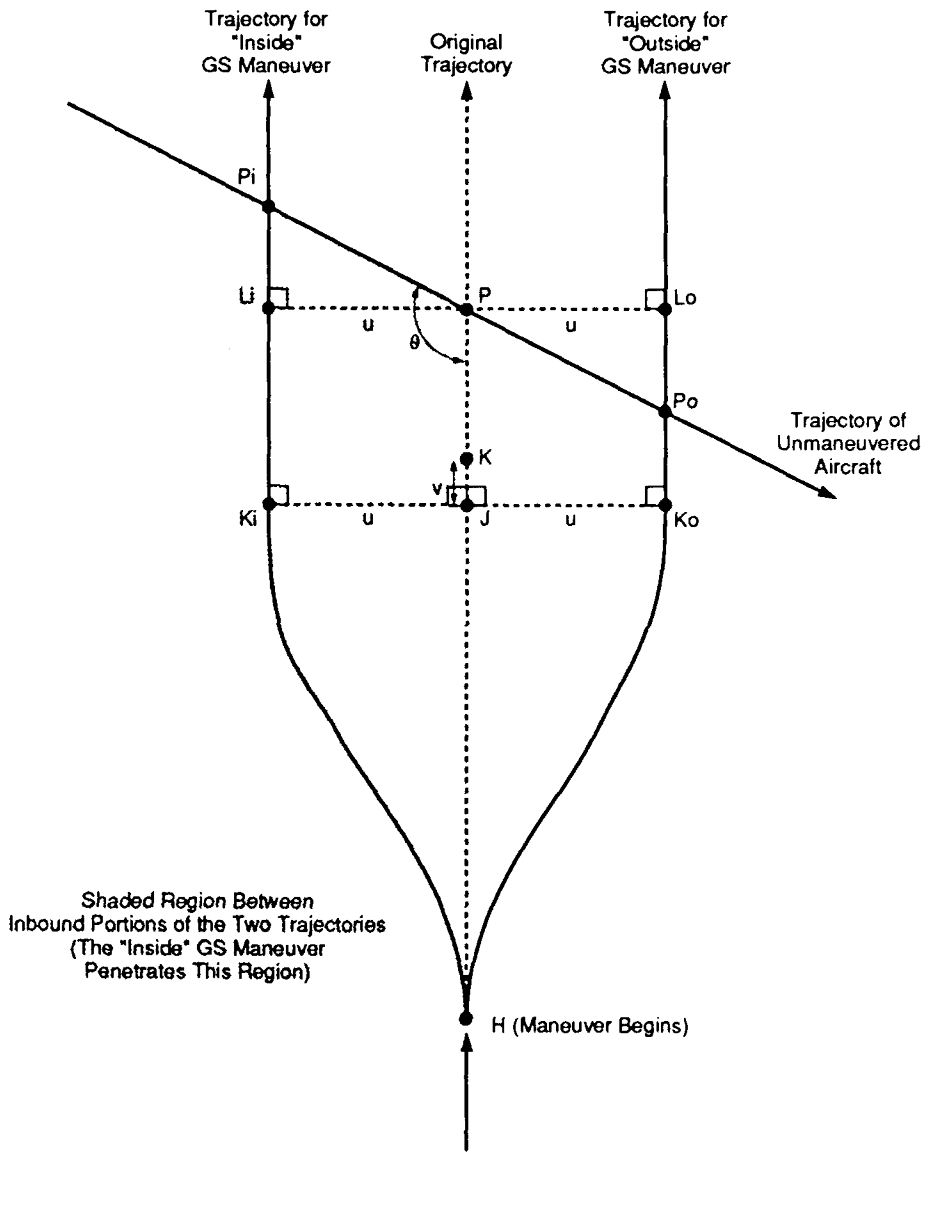

FIGURE 2-3
A TYPICAL GS ENCOUNTER GEOMETRY

2.3 QUANTIZATION OF ONE-ON-ONE LINEAR CONFLICT GEOMETRIES

The state space of horizontally-unaccelerated crossing encounter geometries is parameterized using the following variables:

- θ, the encounter angle (the angle formed by the inbound legs of the two trajectories; angle J-P-P_i in Figure 2-3)
- s, the speed of the faster aircraft
- r, the ratio of the slower aircraft's speed to the faster aircraft's speed (the slower's speed is (rs); r = 1 if the speeds are equal)
- T, the difference between the time the slower aircraft reaches the original intersection P and the time the faster aircraft does so (the faster arrives at time zero; the slower at time T)

The minimum nominal separation between the aircraft (without a maneuver) is a function of θ, s, r and T.

2.4 FASTER AND SLOWER AIRCRAFT

It proves convenient in the GS mathematical derivations to distinguish the two aircraft in a conflict via which is the faster and which is the slower. This distinction allows the relative speed variable r to be defined so that it satisfies $0 < r \leq 1$ (a fact used repeatedly in the derivations). If both aircraft have the same speed, one is arbitrarily designated as "faster"; all results are independent of which is chosen. "Faster" vs. "slower" is used through the GS analysis to distinguish the two aircraft; no other identification terminology is used.

2.5 QUANTIZATION OF GENTLENESS

Gentleness in a GS maneuver is specified via two parameters: the offset u and the induced delay v, as defined in Section 2.2. Both are expressed in units of distance.

In certain applications of GS (generally those involving the MOM shell rather than the ASF shell), bounds are placed a priori on the gentleness parameters. These bounds are denoted uHI and vHI:

$$u \leq uHI \text{ and } v \leq vHI$$

Induced delay v is expressed in units of distance rather than time, because the variable s happens to drop out of many of the GS equations as a result. When the variable T drops out as well (which it does in certain situations), the entire state space of GS geometries (ordinarily specified by four parameters, θ, r, s, and T) becomes two-dimensional (in θ and r). Results covering all geometries can thus be displayed on a single sheet of paper (whose axes are θ and r); this is done for several of the figures in Section 3.

To convert delay distance v into time, of course, one simply divides by the speed (s for the faster aircraft, rs for the slower). For vHI = 2 nmi, for example, the delay is 15 seconds or 20 seconds, respectively, for a 480 kt or 360 kt aircraft.

2.6 PATHKEEPING UNCERTAINTY PARAMETERS

It is useful to quantify the pathkeeping uncertainties for each aircraft as follows:

- Lats, Latf; the plus/minus prediction uncertainties in lateral position (for the slower and faster aircraft, respectively) that accumulate over the interval from H (in Figure 2-2), when the maneuver begins, to the end of the conflict.

- Lons, Lonf; the plus/minus prediction uncertainties in longitudinal position that accumulate over the same interval (for the slower and faster aircraft, respectively).

The four parameters Lats, Latf, Lons, Lonf are expressed in units of distance (nmi).

The two aircraft are not modeled to experience their longitudinal uncertainties (Lons and Lonf) independently. Half of the uncertainty is modeled as independent, while half is modeled to be due to an unpredicted wind vector affecting both aircraft. The idea is that aircraft in a $\theta = 20$- or 30-degree crossing geometry, say, are likely to share an unpredicted headwind or tailwind, rather than one experience a headwind and the other a tailwind. Generally, aircraft in geometries with shallow encounter angles experience somewhat less relative longitudinal uncertainty than aircraft in geometries with large encounter angles.

2.7 INSIDE AND OUTSIDE GS MANEUVERS

To take advantage of symmetry, the two possible turn directions for GS maneuvers are denoted "inside" and "outside" (rather than "left" and "right"). An inside maneuver is one that penetrates the encounter angle θ (the shaded region in Figure 2-3), while an outside maneuver does not. In Figure 2-3, the inside maneuver is H-K_i-L_i-P_i, and the outside maneuver H-K_o-P_o-L_o. The inside-outside notation conveniently allows the (symmetric) mirror image of Figure 2-3 to be treated exactly the same as the actual figure. Left and right are swapped by reflection, but "inside" and "outside" are not. The notation implies that θ need range only between 0 and 180 degrees—a condition that simplifies the analysis.

2.8 LABELING THE FOUR GS MANEUVERS

Using the inside/outside notation, as well as the faster/slower notation, the four maneuvers available to GS (turning either aircraft either direction) are labeled:

- SLOWER OUTSIDE (SO)
- SLOWER INSIDE (SI)
- FASTER OUTSIDE (FO)
- FASTER INSIDE (FI)

2.9 PARAMETERIZING THE SEPARATION ACHIEVED BY GS

Slightly different terminology is used, depending upon how the GS mathematical relationships are to be applied.

2.9.1 ASF-Oriented Applications of GS

For some applications of GS (generally, those involving the ASF shell of AERA 3), a minimum desired separation between aircraft is specified a priori. This is denoted NEEDED_SEP. Gentleness (u, v) is expressed as a function of NEEDED_SEP and the geometry.

The equations relating NEEDED_SEP with the gentleness variables u and v involve the variable T (relative predicted time to intersection). The value of T is accurately known to ASF (a few minutes prior to the conflict) but not to MOM (tens of minutes prior), due to wind uncertainties and other factors.

2.9.2 MOM-Oriented Applications of GS

In other applications of GS (generally those involving the MOM shell), it is more convenient to specify, a priori, bounds on gentleness (uHI, vHI, where $u \leq uHI$ and $v \leq vHI$), and determine the separation achievable by GS within these constraints. This separation, denoted GS_SEP, is expressed as a function of gentleness (uHI, vHI) and the geometry. The set of geometries for which GS_SEP is acceptably large becomes the domain of GS s applicability in AERA 3 for the given uHI and vHI. In geometries where GS_SEP is not acceptably large, separation must be provided in some other way, via longitudinal, vertical, or less-gentle lateral maneuvers.

The equations relating GS_SEP with gentleness bounds uHI and vHI are independent of T. GS_SEP, in fact, depends only on variables that are known well in advance. For a conflict 30 minutes in the future:

- GS_SEP might be 5.5 nmi with an uncertainty of ± 0.4 nmi.
- Predicted closest approach might be 6 nmi with an uncertainty of ± 5 nmi.

The early predictability of GS_SEP is the key to the symbiosis between GS and MOM (see Appendix D). It may not be clear until shortly before the conflict, though, which maneuver for which aircraft (FO, FI, SO, SI) attains GS_SEP separation, or whether a maneuver will be needed at all.

The GS_SEP equations are also independent of s (i.e., they depend on the relative, but not absolute, speeds of the two aircraft).

2.10 ASSURING THAT THE GS MANEUVER IS TIMELY

In addition to seeking resolution maneuvers which are gentle (i.e., have small u and v), GS should also seek (in both its ASF- and MOM-related applications) maneuvers which start only a few minutes prior to the conflict. The main reason for this is that the uncertainty parameters (particularly the longitudinal ones Lons and Lonf) grow as a function of the time interval between the beginning of the maneuver and the conflict. Secondly, for GS to consider long-lookahead resolutions would complicate analysis of the symbiosis between GS and MOM (and perhaps compromise the symbiosis itself). Thirdly, lateral resolutions that start too early simply make no ATC sense. For example, a 10-mile lateral offset maneuver that begins 20 minutes prior to the conflict (e.g., a 5-degree turn at H in Figure 2-3) would have a very small induced delay v, and might appear attractive (considering only the small magnitudes of u and v), but would be inappropriate for GS as applied to AERA 3.

To quantify the intuitive notion that a GS maneuver ought not need to start too early, two bounds are established:

1. The time to achieve the offset itself (time to fly HJ in Figure 2-3) should not be too long.

2. The time to fly from J to the conflict itself should not be too long.

Bound 1, which helps to eliminate the above example that is inappropriate for ATC, can be accomplished effectively (though indirectly) via vLO, a lower bound on v. A suitably chosen lower bound vLO renders GS maneuvers with unacceptably long HJ physically impossible. This indirect bound also proves useful in simplifying the analysis to come: the upper and lower bounds on v, $vLO \leq v \leq vHI$ (which are introduced here for quite different purposes) are easily treated together in formulae.

Bound 2 is important since the GS mathematical relationships assume closest approach occurs after the full lateral offset is achieved (otherwise the GS results would depend upon many additional parameters, including the details of how the offset is attained, which differ for different aircraft).

Bound 2, unlike Bound 1, is difficult to specify a priori in the GS mathematical formulations. Instead, Bound 2 is simply ignored in all the mathematical analyses until the final step (Appendix F,

Sections 11 and 12), in which it is shown (a posteriori) that the achievement of the lateral offset (at J and K) need not occur unreasonably early relative to the conflict itself (see also Appendix E).

The parameter for Bound 1, vLO, thus appears in the GS mathematical relationship given in Section 3, but no such parameter appears for Bound 2.

SECTION 3

OVERVIEW OF THE MATHEMATICAL RELATIONSHIPS AMONG THE GS PARAMETERS

This section presents (without proof) the mathematical relationships among the various parameters (as defined in Section 2). Some of the relationships are illustrated as figures. The proofs are given in Appendix F.

The parameters are:

(a) The encounter geometry (θ, r, s, T)

(b) The uncertainty in the aircraft predicted positions during the conflict ("strictness" by which the aircraft follows its plan) (Lonf, Lons, Latf, Lats)

(c) The separation between the aircraft due to a GS maneuver (NEEDED_SEP or GS_SEP)

(d) The "gentleness" of the resolution maneuver, as parameterized by:

- Magnitude of the parallel lateral offset (u, or uHI)
- Delay induced by the parallel lateral offset (v, or vHI and vLO)

A MOM-oriented relationship is derived, in which one solves for GS_SEP as a function of θ, r, uHI, vHI, vLO, and the four uncertainty parameters (all nine are known well in advance). GS_SEP is the amount of separation GS can provide for the given geometry, given that GS's maneuvers must meet the specified gentleness criteria. The set of geometries for which GS_SEP is acceptable forms the domain of GS's applicability to MOM (for the given uHI, vHI, vLO).

Also, an ASF-oriented relationship is derived, in which one solves for gentleness (u and v), as a function of θ, r, s, T, NEEDED_SEP, and the four uncertainty parameters.

Finally, an expression is derived to determine which of the four GS maneuvers provides the most separation between the aircraft. The output is a selection of one of the four maneuvers (SO, SI, FO, FI), chosen as a function of θ, r, s, T, uHI, vHI, and vLO (it is independent of the uncertainty parameters). For an application using u and v, the same expression can be used, substituting u = uHI, and v = vHI = vLO.

3.1 MOM-ORIENTED APPLICATION OF GS: SOLVING FOR GS_SEP

In this section, a procedure is given to determine the separation achievable (GS_SEP), as a function of θ, r, uHI, vHI and vLO, and the uncertainty parameters.

The state-space (of possible values of θ, r, uHI, vHI, and vLO) is first partitioned into three disjoint subspaces; an expression for GS_SEP is then given for each subspace.

The three subspaces are known as "SO v FI", "SI v FI", and "FO v FI". Within the subspace "SO v FI", one can say the following:

> "I know that, among the four possible GS maneuvers, SO or FI provides the most separation. SO causes the faster aircraft to reach the new intersection first, while FI causes the slower aircraft to reach it first. Without knowing the value of T (not known accurately in MOM's timeframe), I cannot tell which of SO or FI provides more separation."

Within the other two subspaces, the same applies if "SI" or "FO" is substituted for "SO".

To determine which subspace a given set of values for (θ, r, uHI, vHI, vLO) falls within, consider the following three inequalities:

1. $\cos\theta > r$
2. $uHI\,(1 + r)\,(1 - \cos\theta) > (vHI + (r\ vLO))\sin\theta$
3. $uHI\,(1 - r)\,(1 + \cos\theta) > (vHI + (r\ vLO))\sin\theta$

If 2 and 3 both hold, the subspace is "FO v FI". Otherwise, if 1 holds, the subspace is "SO v FI". Otherwise, the subspace is "SI v FI".

The following expressions determine GS_SEP for each subspace:

Subspace "SO v FI": $GS\_SEP = (\,uHI\ (1 - r)(1 + \cos\theta) + (1+ r)\ \sin\theta\,(vHI\quad) - 2X)\,/\,(2Y)$
Subspace "SI v FI": $GS\_SEP = (\,uHI\ (1 + r)(1 - \cos\theta) + (1+ r)\ \sin\theta\,(vHI\quad) - 2X)\,/\,(2Y)$
Subspace "FO v FI": $GS\_SEP = (2uHI\quad (1 - r\cos\theta) + (\quad r)\ \sin\theta\,(vHI-vLO) - 2X)\,/\,(2Y)$

where:

$X = (Lons + (r\ Lonf))\sin\theta\,(1 + \sin(\theta/2)/2) + Lats\,|\cos\theta - r| + Latf\,(1 - (r\cos\theta))$, and
$Y = SQRT(r^2 - 2r\cos\theta + 1)$

The term X reflects the contribution (i.e., detriment) to GS_SEP due to the four uncertainty terms. The contribution (detriment) due to any individual uncertainty term can be easily determined from the expression for X—for instance, that of Latf is simply $-(1-(r \cos\theta)) / Y$.

Figure 3-1 shows GS_SEP as a function of r and θ, assuming uHI = 12 nmi, vHI = vLO = 2.0 nmi, Lats = Latf = 0.5 nmi, and Lons = Lonf = 1.0 nmi. Note that GS_SEP exceeds 5 nmi except in the shaded region.

Figure 3-2 shows the partitioning of (r, θ) into subspaces, for the above values of uHI, vHI, and vLO.

3.2 ASF-ORIENTED APPLICATION OF GS: SOLVING FOR u AND v

The following equations may be used to determine the gentlest GS maneuver (as parameterized by u and v) which provides some minimum desired separation, denoted NEEDED_SEP. The values of u and v here are not bounded by uHI, vHI, and vLO; they simply are as large as necessary (and as small as possible) to guarantee NEEDED_SEP separation.

One simply solves for u and v using one of the four equations below, each applicable to one of the four GS maneuvers:

SO: $\text{NEEDED\_SEP} = \text{ABS}\ (-(ru) + (u \cos\theta) + (v \sin\theta) + (rsT \sin\theta) - X) / Y$

SI: $\text{NEEDED\_SEP} = \text{ABS}\ (+(ru) - (u \cos\theta) + (v \sin\theta) + (rsT \sin\theta) - X) / Y$

FO: $\text{NEEDED\_SEP} = \text{ABS}\ (+(u) - (ru \cos\theta) - (rv \sin\theta) + (rsT \sin\theta) - X) / Y$

FI: $\text{NEEDED\_SEP} = \text{ABS}\ (-(u) + (ru \cos\theta) - (rv \sin\theta) + (rsT \sin\theta) - X) / Y$

where X and Y are as defined in the previous section.

Note that (for a given GS maneuver) there is one equation for two variables (u and v), so there are many solutions. One would prefer solutions, however, which are realistic for ATC. If one desires, say, to keep the ratio v/u constant (see Figure 2-2), an equation such as (v = 0.2 u) can be used as a second equation to determine a particular u and v. Alternatively, if it is desired to keep distance HJ constant (and one is willing, for simplicity, to assume the aircraft travels straight from H to K_O so that $HK_O = HJ + v$), the second equation could be $u^2 + (HJ)^2 = (HJ+v)^2$. There are other possibilities as well to relate u and v.

Figure 3-3 shows, as a function of r and θ, values of (u, v) which provide NEEDED_SEP = 5 nmi of separation, using (as in Figure 3-1) Lats = Latf = 0.5 nmi, and Lons = Lonf = 1.0 nmi. Note that the shaded region in the upper right corner of Figure 3-3 is identical to the shaded region in Figure 3-1; at the boundary of this common shaded region NEEDED_SEP = GS_SEP = 5 nmi, u = uHI = 12 nmi, and v = vLO = vHI = 2 nmi.

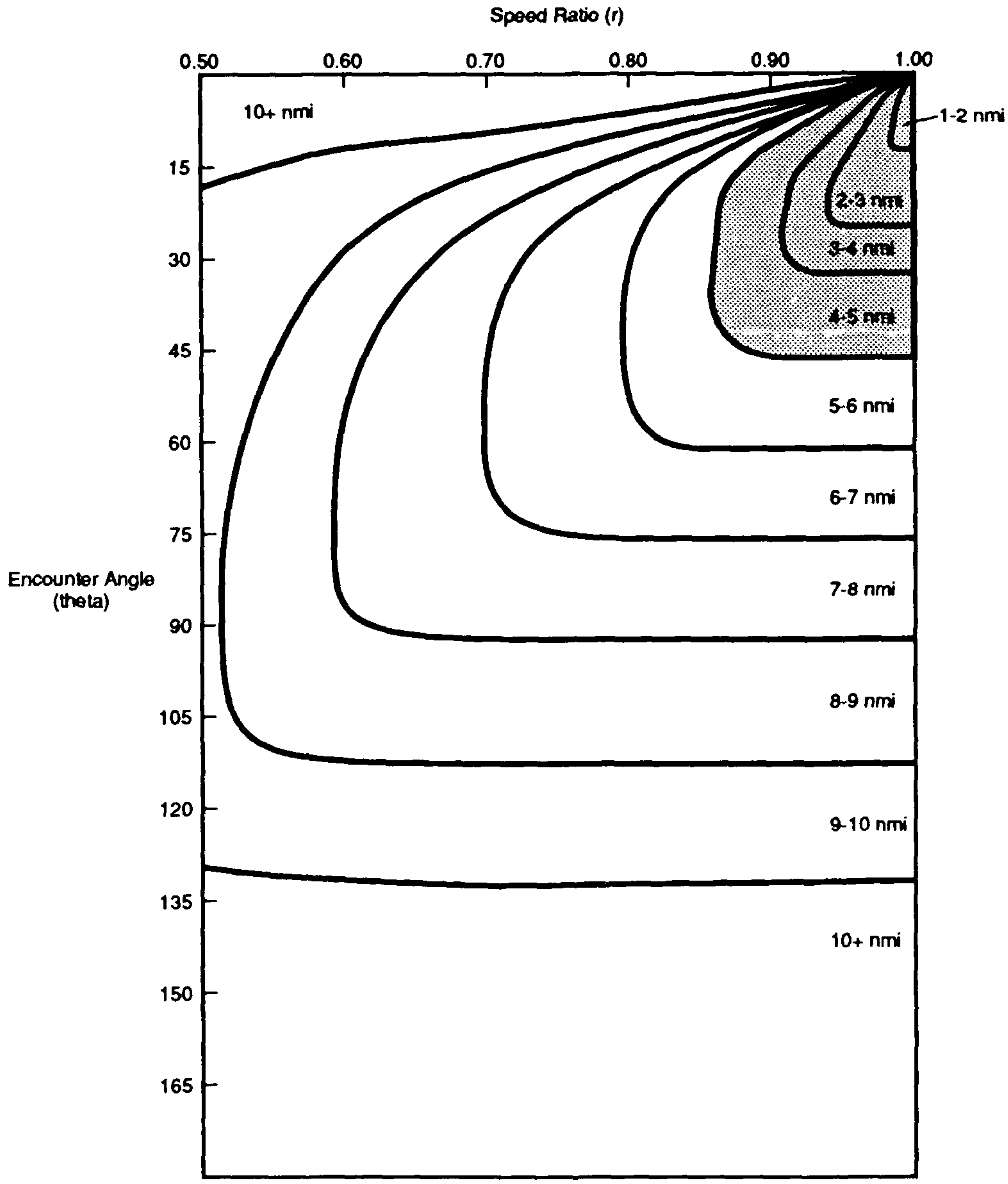

FIGURE 3-1

SEPARATION ACHIEVED (GS_SEP), AS A FUNCTION OF ENCOUNTER GEOMETRY (r, THETA), GIVEN THAT THE MANEUVERS SATISFY GENTLENESS CRITERIA (OFFSET u≤12 nmi, DELAY v≤2 nmi); FIVE MILES SEPARATION IS PROVIDED FOR ALL GEOMETRIES EXCEPT IN SHADED REGION

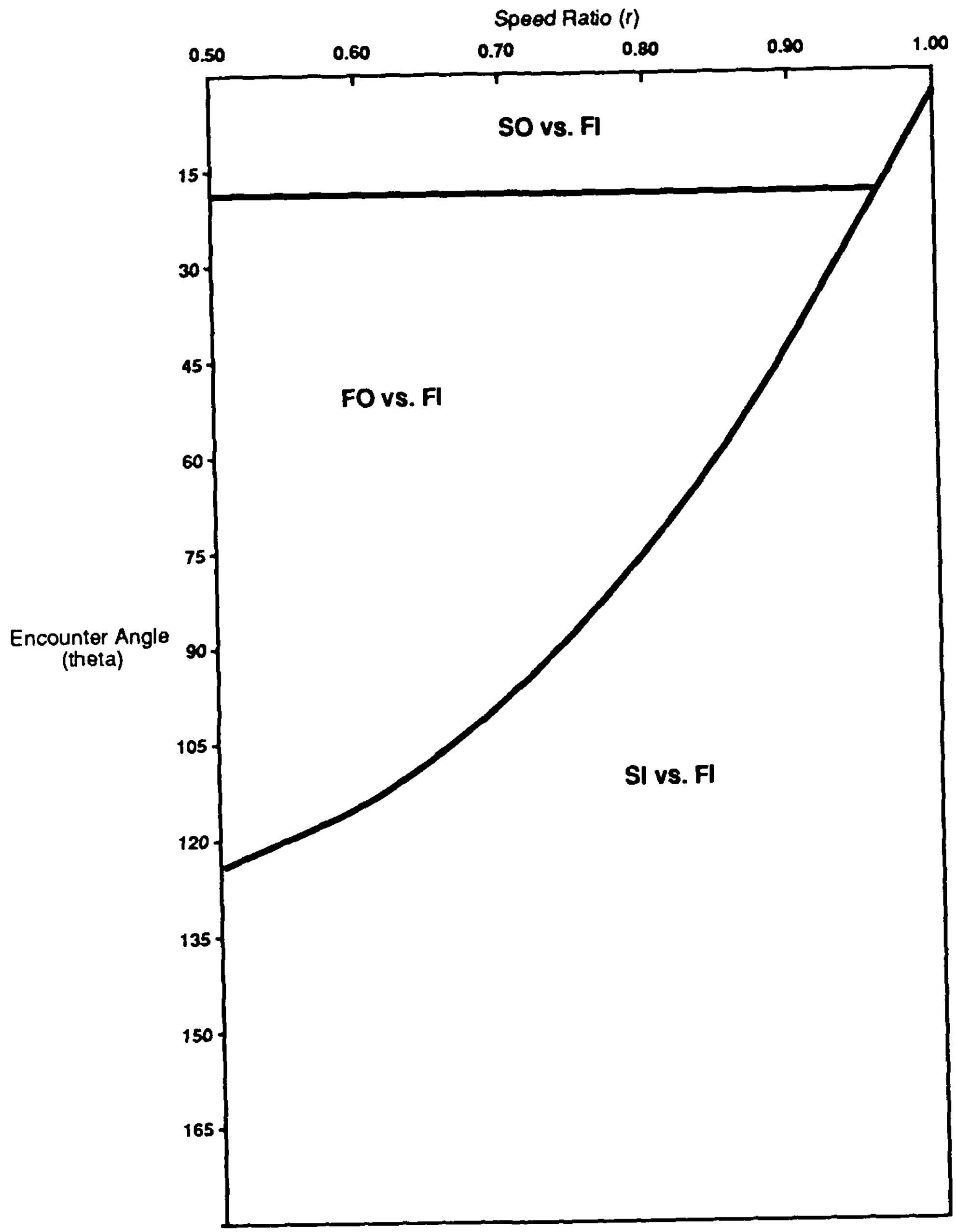

FIGURE 3-2
DIVISION OF ENCOUNTER GEOMETRY STATE SPACE (R, THETA) INTO THREE SUBSPACES

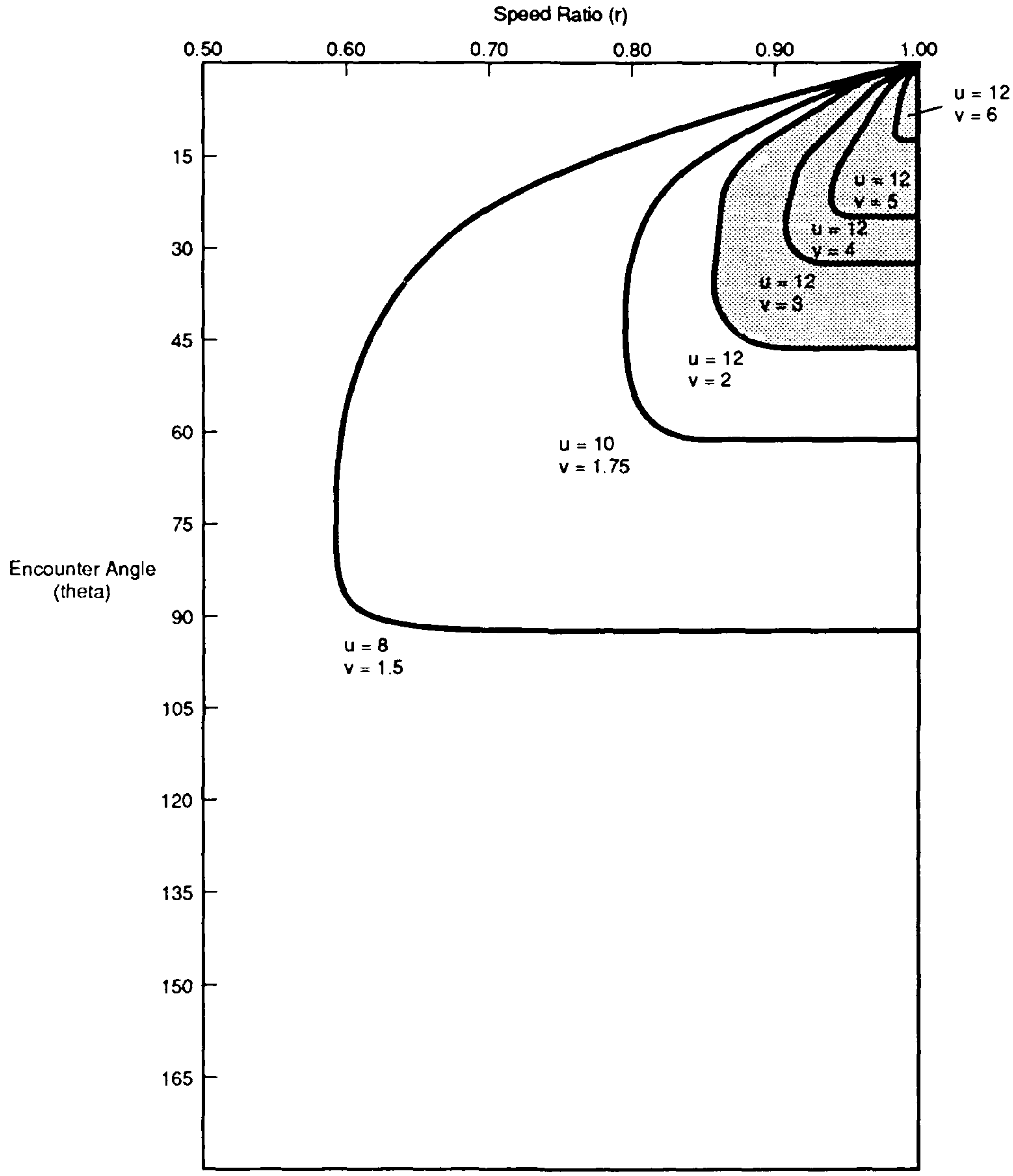

FIGURE 3-3
GENTLENESS (OFFSET u, DELAY v) AS A FUNCTION OF ENCOUNTER GEOMETRY (r, THETA), GIVEN THAT FIVE MILES SEPARATION IS REQUIRED

3.3 DETERMINING WHICH OF THE FOUR GS MANEUVERS PROVIDES THE MOST SEPARATION

In this section, an expression is derived to determine which of the four GS maneuvers provides the most separation between the aircraft. The output is a selection of one of the four maneuvers (SO, SI, FO, FI), chosen as a function of θ, r, s, T, uHI, vHI, vLO (the choice is independent of the uncertainty parameters). For an application using u and v (rather than uHI, vHI, vLO), the expression given here can also be used, substituting u = uHI, and v = vHI = vLO.

The expression involves the three subspaces "SO v FI", "SI v FI", and "FO v FI," as computed in Section 3.1. Here, however, the variable T (the relative timing of the two aircraft to the original intersection) is assumed to be known. T is used to determine which of the following two possible GS strategies provides the most separation:

– Separate the aircraft via having the faster reach the new intersection first (known as GET FASTER AHEAD)

– Separate the aircraft via having the slower reach the new intersection first (known as GET SLOWER AHEAD)

GET FASTER AHEAD provides more separation than GET SLOWER AHEAD if and only if (as a function of subspace):

Subspace "SO v FI": $T < (uHI\,(1+r)\,(1-\cos\theta)\ -\ (1-r)\ \sin\theta\,(vHI\ ))\ /\ (2r\ s\ \sin\theta)$

Subspace "SI v FI": $T < (uHI\,(1-r)\,(1+\cos\theta)\ -\ (1-r)\ \sin\theta\,(vHI\ ))\ /\ (2r\ s\ \sin\theta)$

Subspace "FO v FI": $T < \qquad (+\ \ (\ r)\ \sin\theta\,(vHI+vLO\ ))\ /\ (2r\ s\ \sin\theta)$

If GET SLOWER AHEAD is selected, the best GS maneuver is FI (for any subspace).

If GET FASTER AHEAD is selected, the best GS maneuver is SO, SI, or FO, depending upon the subspace (see Figure 3-2).

SECTION 4

SOME SUGGESTED FURTHER ANALYSES

The GS results documented here show that gentle resolutions can resolve large classes of pairwise encounter geometries with a high degree of predictability. However, the GS analysis is based on several assumptions:

(a) there are no unexpectedly large deviations from trajectory,
(b) no third aircraft are laterally near the conflict, and
(c) the aircraft have no horizontal accelerations in the vicinity of the conflict.

To help fill in remaining gaps in the verification of AERA 3 as a safe provider of improved ATC services, one would like to extend GS-like results to more general circumstances, in which (a), (b), and (c) do not apply. The following sections discuss the ability to generalize GS in this fashion. As shall be seen, assumption (a) is deeply built into the GS approach; relaxation of this assumption will require a different approach. However, the GS analysis may be extended in certain circumstances to allow relaxations of (b) and (c).

Sections 4.1, 4.2, and 4.3 deal, respectively, with relaxing the three assumptions.

4.1 UNEXPECTED DEVIATIONS FROM TRAJECTORY

The GS parameters reflecting pathkeeping uncertainty can be made large enough to include routine uncertainty, as well as a margin to allow for cases where an aircraft unintentionally drifts off its path, is so notified by AERA 3, and complies with an AERA 3 directive to return to path.

The pathkeeping uncertainty parameters cannot realistically be enlarged to reflect all possible future deviations from path, e.g., due to communication or equipment failure or other emergency. Such situations are a fact of life that AERA 3 must deal with safely (and without recourse to last-minute human help). There is no need, however, for AERA 3 to consider only gentle maneuvers in these situations. The AERA 3 paradigm is to provide:

- Safety, and a high degree of benefits (to public/airlines/pilots), in routine circumstances.

- Safety, in emergencies or non-routine circumstances.

Dealing with emergencies is a key part of AERA 3 verification; work on this topic is underway at MITRE. However, GS is not expected to play any role here.

4.2 PAIRWISE CONFLICTS WITH THIRD AIRCRAFT NEARBY

Requiring assumption (b) (*no third aircraft or other obstacle in the vicinity of pairwise conflicts*) may not be possible or practical in relatively high density traffic situations (dealing with such situations is claimed to be an AERA 3 benefit).

In certain circumstances, it is possible to extend GS results to situations where (b) does not apply. Several possible extensions are discussed in Appendix B. These generally involve determining what subset of the four GS maneuvers (SO, SI, FO, FI) are not available due to the third aircraft, and then determining, as a function of (r, θ), how much separation is achievable using the maneuvers that are available.

Fortunately, (b) is not necessary for safety—only for the benefit of applying GS resolution maneuvers. What is necessary for safety is that at least one out (e.g., safe lateral, longitudinal, or vertical maneuver) be available for each conflict, at the time that a resolution is needed. This assurance of outs is one of the key roles planned for MOM. MOM's outs are not always gentle in the GS sense, but are nevertheless routine ATC maneuvers, generally of comparable magnitude to those used today.

Hence, if AERA 3 could resolve, say, 90 percent of the conflicts with a GS maneuver (or something better, if available), but is forced to fall back on a MOM out for the other 10 percent of the conflicts, AERA 3 can still be said to provide safety and guarantee a high degree of user benefits routinely.

Of course, the verification burden is passed along to MOM to assure outs for each possible conflict. MOM, in turn, (see Appendix C and [5]) passes some of the burden along to AERA 3's non-automated outer shell functions with an even longer lookahead time. These functions prevent overall traffic density from becoming too high (no outs will be available to MOM if traffic is too dense). Ultimately, some limit on traffic density must be assumed, in order to verify AERA 3.

To summarize, assumption (b) is not a rigid one. It may be safely waived for particular conflicts, with only a minimum degradation of user benefits, as long as MOM can always be relied upon to provide an out. As (b) is waived more frequently, GS-derived guarantees on user benefits gradually diminish.

4.3 HORIZONTAL ACCELERATIONS IN THE VICINITY OF THE CONFLICT

The burden of verifying AERA 3 is placed primarily on AERA 3's emergency-handling functions when (a) is relaxed, and on the MOM function when (b) is relaxed. Additional GS analysis must continue to assume (a) and probably can make only limited contributions in relaxing (b). However, there appear to be several possible approaches to extend GS analysis to situations where (c) is relaxed—i.e., where the trajectories of the aircraft may in fact have horizontal accelerations in the vicinity of the conflict. These possible approaches are outlined in Appendix B. Generally, these involve modifying the trajectories so that the horizontal accelerations occur before or after, but not

during, the conflict. It is far from clear whether or how these approaches can be consolidated into a rigorous theoretical result; the material is presented only as suggestions for further GS-related work.

One interesting feature apparent in Figure 3-1 is that GS_SEP, the separation guaranteed by GS, grows as a function of the relative velocity between the two aircraft. The relative velocity, which is zero at the upper right corner of the figure ($r = 1$, $\theta = 0$), grows as one moves leftward and downward in the figure. GS_SEP does likewise. As relative velocity is, in some sense, a measure of the "severity" of a conflict, GS might be said to do best when it is most needed, and to do least well in the least severe geometries. If it is decided to make separation standards vary as a function of the encounter geometry, it may well turn out that the separation standards resemble those given for GS_SEP in Figure 3-1.

SECTION 5

SUMMARY AND CONCLUSIONS

An algorithm, called Gentle-Strict (GS), is given for automated resolution of crossing conflicts between two aircraft. The performance of GS is evaluated in terms of minimum guaranteed separation for each pairwise crossing geometry.

GS is part of a larger effort, sponsored by the FAA, to automate many ATC functions. This effort is the third phase of a project called AERA 3. The major goals of this larger effort include provision of increased ATC services, capacity, and safety. It is assumed that the tactical detection and resolution of aircraft-aircraft conflicts will be completely automated. Humans will not be involved in real time-critical separation assurance decisions, but will participate in planning.

GS resolutions typically entail:

(a) a gentle lateral maneuver for one aircraft, and
(b) a strictly-monitored directive to the other aircraft to avoid deviations from centerline in the "unsafe" direction.

When adequate separation will exist barring lateral deviations from nominal by either aircraft, (b) can be given to both aircraft; ATC users benefit since neither aircraft must maneuver.

GS considers only parallel lateral offset resolution maneuvers. These maneuvers are parameterized by the magnitude of the lateral offset (denoted u), and the induced delay upon the aircraft (denoted v); both u and v are measured in nmi (although the delay is often converted into units of time for presentation of data).

The gentleness of a maneuver is parameterized also by u and v. In certain applications, upper bounds (uHI and vHI) are specified a priori for u and v.

For uHI = 12 nmi, and vHI = 2 nmi (equivalent to a delay of 15 to 20 seconds at typical en route speeds), GS provides a net of 5 nmi guaranteed separation for all crossing geometries where θ is at least 47 degrees, or the relative speed (r) of the slower to the faster aircraft is less than 0.84. Upper bounds on u and v which provide 5 nmi guaranteed separation can be calculated for any value of θ and r.

Notable features of the GS algorithm include:

- Simplicity: The concepts are readily explained and understood either in overview or in full detail; coding requires only a few hundred lines.

- Geometric basis: The state space of encounter geometries is fully parameterized; no recourse is made to human expertise.

- Built-in quantitative limit on downstream trajectory impact: Stability of planning is enhanced by the fact that resolutions to individual conflicts do not take an aircraft too far off its original route; user benefits (fuel/time savings) also result.

GS is not proposed as the one-and-only means for AERA 3 to generate resolutions for pairwise conflicts. Rather, GS will serve as a baseline or default algorithm, about which mathematically-precise statements can be made. These, it is hoped, will ultimately lead to verification strategies for AERA 3.

Non-GS maneuvers, such as vertical resolutions (not considered by GS) may be best in some situations. Such maneuvers, however, would have to be checked in each instance for safety and for compatibility with the planning functions, and used on an opportunistic basis only. In contrast, safety and compatibility with the planning functions is built in, a priori, to the class of GS maneuvers.

APPENDIX A

ROLE OF HUMAN EXPERTISE IN AUTOMATED ATC

The role of human expertise in automated ATC is discussed briefly in Section 1.3, and more fully here. Specifically, the question is addressed:

- Why propose a resolution algorithm (like GS) that essentially starts from scratch, ignoring the expertise that controllers have built up over the years?

Previous research into automated selection of resolutions for aircraft-aircraft conflicts [3] has centered upon applying the expertise of current air traffic controllers (as modified somewhat to anticipate automation) to a knowledge base with a complex set of rules. The automation is assumed to suggest one or more resolutions to the controller, who then makes the final decision.

This appendix exposes the justification for not applying this large (but difficult to quantize) body of knowledge, which has generally worked well for many years. The heart of the matter is that the automation of time-critical decisions profoundly changes the nature of the conflict resolution problem.

A.1 AUTOMATION CHANGES THE CONSTRAINTS ON CONFLICT RESOLUTION

Controllers separating aircraft today address a variety of diverse subgoals. Some of the subgoals carry over to the highly-automated ATC envisioned here (e.g., avoiding violations of separation standards , avoiding unnecessary maneuvers, etc.), but some would not, such as:

- Ensuring the resolution follows current operational procedures. (These may change as a result of automation.)

- Ensuring that the controller quickly grasps how the resolution will provide separation (based on patterns familiar or obvious to him/her).

- Ensuring that the resolution does not require continual or time-consuming monitoring by the controller.

It is a difficult enough problem (in artificial intelligence) to express controller expertise in an automated knowledge base; it appears considerably more difficult to automate exactly those features motivated by certain subgoals rather than others. To make the task even more difficult, some specific types of expertise reflect both sets of subgoals.

In addition:

- Separation standards may change. For instance, today's 5 nmi horizontal separation standard is fixed for all geometries (controllers cannot be expected to calculate a mathematically complex separation standard in a time-critical environment). The automation may be able to use a separation standard based purely on considerations of safety (rather than on human limitations). Overtakes, for example, may require less separation than head-ons.

- Uncertainties in predicted positions may be substantially reduced, over the next decade, due to improved wind measurements, downlinking of aircraft-derived data, and changes in operational procedures. These factors may imply not only quantitative, but also qualitative, changes in transitioning from manual to automated conflict resolution strategies.

To be sure, many of the factors discussed above are already important in the development of AERA 1 and AERA 2. For instance, MITRE engineers have been holding an ongoing series of consultations with a panel of expert controllers to evaluate the impact of possible changes in ATC procedures, separation standards, accuracies of data, aircraft equipage, etc., upon AERA 1 and AERA 2.

The key point here is that for AERA 3, such changes (particularly in operational procedures) will be significantly greater. While current controller expertise (modified where appropriate to account for automation) remains a good starting basis for earlier versions of AERA, automation (modified where appropriate to apply controller expertise) becomes the starting basis for AERA 3.

A.2 VERIFICATION ISSUES

An important issue influencing GS design is verification. In particular, any software needed for automated ATC must be certified as safe to use.

Until AERA 3, the certification process has relied heavily upon the fact that, after all, the human controller retains the ultimate responsibility for separation assurance. An automation tool that is based on controller expertise promotes a symbiosis between the controller and the computer; they speak each other's language. This symbiosis facilitates the verification process.

With the controller out of the time-critical loop, as in AERA 3, these verification benefits of expertise-based algorithms are diminished.

In an automated environment, the better path to verification may be to consider algorithms whose foundation rests upon mathematics rather than upon human expertise, for reasons including the following:

- Mathematics is an unambiguous language; the state space addressed (by a math-oriented resolution algorithm such as GS), and the variables that span it, may be precisely defined

(as in Section 2 for GS). (In contrast, controllers often disagree among themselves about which resolution is best. As practiced by controllers today, conflict resolution is something of an art.)

- The reasons for the algorithm's choices (in the case of GS) are accessible to anyone with a knowledge of simple algebra.

- There appear to be benefits as well in keeping the resolution algorithm as simple as possible, as long as it safely separates the aircraft, and more generally, meets the four major ATC goals listed in Section 1.1 (increased services, capacity, safety, productivity). Experience has shown that the difficulty of certifying software in general rises dramatically with complexity (as measured, say, by lines of code required to implement; GS, for example, takes less than 100 lines of Fortran code).

- Mathematical proof is a powerful technique for verification. One of the goals of GS is to facilitate such a proof. Even if a complete proof cannot be achieved, there are benefits to be achieved by trying. For instance, simulation testing could focus extensively on exactly those parts of the algorithm that cannot be proved.

APPENDIX B

POSSIBLE STRATEGIES TO EXTEND GS-LIKE RESULTS TO ACCELERATED GEOMETRIES

All results in this document (outside this appendix) make the following assumptions about the two aircraft, in the immediate vicinity of their conflict:

(a) They follow their predicted trajectories within a specified tolerance.

(b) They are free of interactions with third aircraft or other obstacles.

(c) They have trajectories that are free of horizontal accelerations.

It is important for the verification of AERA 3 to extend GS-like results to situations in which (a), (b), and (c) do not apply. In Section 1.3, ideas are outlined for future GS work which may help relax assumptions (b) and (particularly) (c). In this appendix, these ideas are explored in more detail. It is far from clear whether or how these ideas can be consolidated into a proof; they are presented only as suggestions for future GS work.

B.1 RELAXING ASSUMPTION (c)

Consider Figure B-1. Two aircraft, originally crossing at 90 degrees, are rerouted around a storm. In Figure B-1a, the reroutes are such that GS guarantees are inapplicable, for assumption (c) is violated. Figures B-1b through B-1d show alternative reroutes that allow GS guarantees.

In Figure B-1b, the route bends are (deliberately) placed away from the route intersection, so as to allow GS results to apply. GS chooses among its usual four standard maneuvers SO, SI, FO, FI, as discussed in Section 2.2, and shown (via four small bent arrows) in Figure B-1b. The basic idea is that the long lookahead planning function very often has a wide degree of choice in selecting maneuvers, and can often avoid compromising GS guarantees with minimal effort.

In Figure B-1c, slightly different reroutes result in route bends prior to the intersection for both aircraft. Although (c) is violated, a simple modification to Equation (12) in Appendix F allows GS_SEP to be computed, using four new types of GS-like maneuvers (shown, again, as four small arrows). Instead of each aircraft having possible INSIDE and OUTSIDE GS maneuvers (as defined in Section 2.2), each aircraft has the possibility of executing its turn (of ALPHA or BETA degrees) EARLY or LATE. (Equation 12 calculates the change, as a result of a GS maneuver, in the relative time that the two aircraft arrive at the new versus the old intersection; this result can be made to be a function of the route bend angles, ALPHA modified calculation of GS_SEP can be made long in advance, just as can a standard calculation of GS_SEP when (c) is satisfied.)

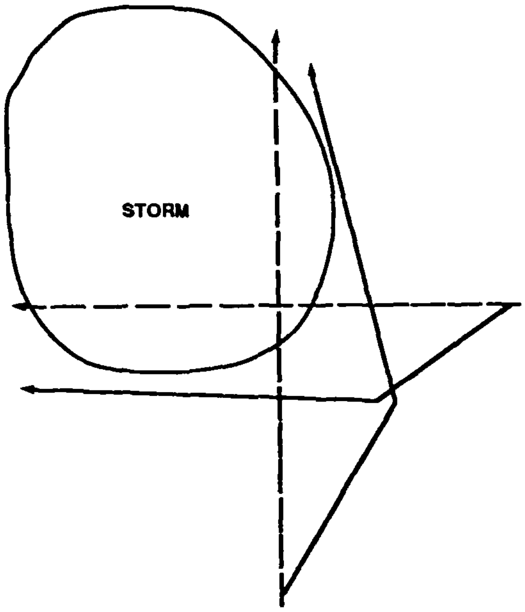

FIGURE B-1a
GS DOES NOT APPLY; ROUTE BEND OCCURS TOO NEAR CONFLICT

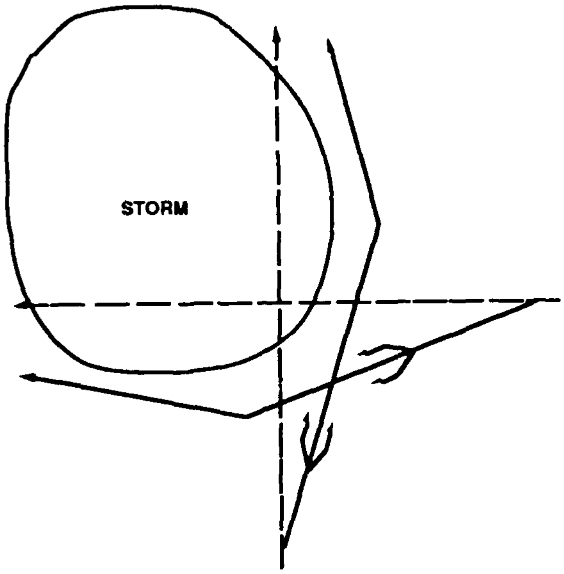

FIGURE B-1b
GS APPLIES; ROUTE BENDS OCCUR BEYOND CONFLICT

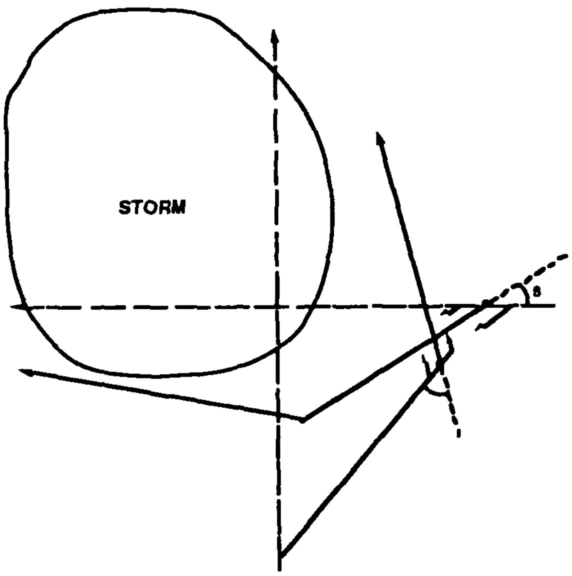

FIGURE B-1c
A MODIFIED VERSION OF GS APPLIES

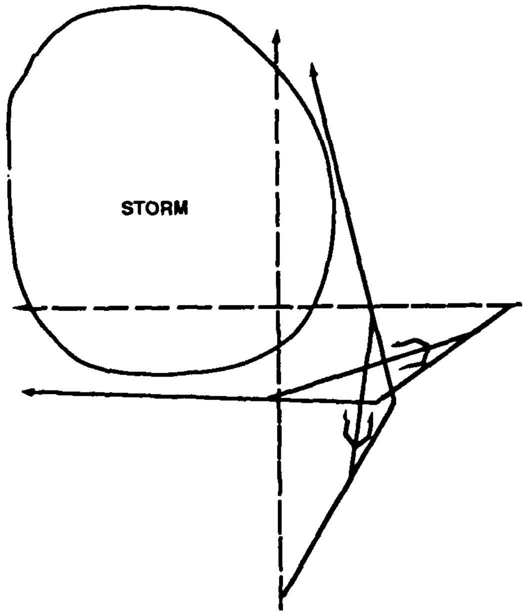

FIGURE B-1d
GS, VIA LOCAL-ONLY CHANGE TO FIGURE B-1a GEOMETRY, APPLIES

The sensitivity of GS_SEP to ALPHA and BETA has not yet been calculated. This is a possible direction for future GS work.

In Figure B-1d, it is assumed that (for whatever reason) AERA 3 has rerouted the aircraft as in Figure B-1a (nullifying GS guarantees), and that (due to dense traffic), significant changes to the two aircraft's reroutes (such as altering them to resemble Figures B-1b and B-1c) cannot be made. However, a slight straightening of the two routes near the conflict may be possible, as shown in Figure B-1d (indeed, MOM knows long in advance whether this is possible). If so, GS can then be applied. (It is true that the pilots might get a bit annoyed by the many small route bends, were both a straightening and a GS solution actually implemented in this situation. However, AERA 3's one-on-one resolution algorithm would probably find a simpler solution. The value of establishing a GS solution well in advance lies in verification, and on establishing a lower bound on the gentleness of the maneuver needed.)

Other possible reasons for horizontal acceleration that violate (c) include:

1. They are part of a solution to a long-term problem (e.g., metering or density).

In this case, the solution to the long-term problem may be delayable until after the conflict, so that GS can apply.

2. They are necessary for navigation (e.g., route bends at VORs).

This case is expected to be rare in the AERA 3 timeframe; most aircraft at en route altitudes will be able to navigate from point to point.

(If aircraft cannot so navigate, they cannot, in general, perform GS maneuvers in the first place.)

3. They are motivated by user benefits (independent of traffic, storms, etc.).

Instances where this case impairs GS applicability would appear to be quite rare. Over long distances, the location of jet streams and other weather features sometimes causes optimal routes to bend, but the exact timing of the bends is not critical.

B.2 RELAXING ASSUMPTION (b)

If (b) does not hold—that is, a third aircraft or other obstacle such as a storm is near the intersection, GS guarantees do not in general apply. "Near" means within distance (uHI + buffer) left or right of either aircraft, where uHI is the maximum lateral offset and "buffer" is the closest distance that two aircraft are allowed to approach, plus an allowance for uncertainty.

In some cases, predictable long in advance, GS guarantees do apply despite the violation of (b). A third aircraft, for instance, might rule out one or two of the four GS maneuvers (SI, SO, FI, FO), but not the two between which GS must choose (as outlined in Appendix F, Sections F.7 through F.8).

Or, a third aircraft might rule out one of the two GS maneuvers judged to be best for the two strategies GSA (get slower aircraft ahead) and GFA (get faster aircraft ahead), but the second-best maneuver for GSA and/or GFA may do almost as well, or adequately well. Such cases have not yet been analyzed in detail. One useful approach might be to see how well GS does (as a function of r and q) when a third aircraft denies GS the use of one of (SI, SO), and/or of one of (FI, FO). For example, it may be that for a particular r and θ:

- When restricted to SI and FI, GS guarantees 7.2 nmi
- When restricted to SO and FI, GS guarantees 6.2 nmi
- When restricted to SI and FO, GS guarantees 4.5 nmi
- When restricted to SO and FO, GS guarantees 5.7 nmi

Then for that (r, θ), GS would guarantee 4.5 nmi separation despite the existence of a third aircraft that rules out one maneuver for each aircraft. Also, GS would guarantee min (max [4.5, 5.7], max [7.2, 6.2]) = 5.7 nmi separation, if, say, a third aircraft interferes with one of the faster aircraft's two maneuvers (unspecified whether FO or FI), but with neither of the slower aircraft's maneuvers. The inner "max's" reflect the free choice of SI vs. SO, while the outer "min" reflects the uncertainty of not knowing which of (FI, FO) will be available for use by GS.

B.3 HELP FROM MOM

Just as MOM works to assure that no complex problems remain to be solved on a short time scale, it may also help enable GS benefits. There are typically many ways to simplify complex problems, and MOM may in many instances be able to pick a simplification that has the additional benefit of allowing GS solutions for all remaining (one-on-one) possible conflicts. It may favor earmarked maneuvers (see Appendix C) specifically for possible conflicts that do not have GS solutions, leaving only ones that do.

B.4 CONCLUSION

Many ideas to extend GS work to situations where (b) or (c) above have been presented. As mentioned above, these ideas are presented only as suggestions for future GS and AERA 3 work.

APPENDIX C

OVERVIEW OF MANEUVER OPTION MANAGER IN THE CONTEXT OF AERA 3

One of the key applications of the GS methodology is to AERA 3's Maneuver Option Manager (MOM)[5]. This appendix provides an overview of MOM and those parts of AERA 3 with which MOM interacts closely. The assesses symbiosis developed between MOM and GS, outlined in Section 1, is explored in more detail in Appendix D.

In AERA 3 the automation assumes, for the first time, the responsibility of separating aircraft. AERA 3 is expected to allow denser traffic and to provide increased services to users than is possible under prior phases of ATC automation.

The human retains a role in formulating long-term strategies and traffic flow patterns, but does not in general deal with individual aircraft. The human (for the first time) is no longer involved in routine or time-critical separation assurance decisions.

AERA 3 has a hierarchy of functions that can be compared to nested shells; each shell relies upon the next-outer shell to assure that it is not given a problem too difficult to handle.

C.1 AERA 3 OUTER SHELL

AERA 3's outermost shell is known as the Airspace Manager Planning Functions. These functions are automation aids to be used by a human (known as the Airspace Manager). This shell is loosely analogous to today's local flow management, but includes some new capabilities and is currently under development at MITRE.

The Airspace Manager Planning Functions generally deal with aircraft on an aggregate or statistical basis. The lookahead period, which extends out to perhaps 90 minutes, is large enough that prediction uncertainties preclude detailed analysis of specific aircraft-to-aircraft interactions.

Among the planning functions in this outer shell for AERA 3, Density/Complexity Manager has the closest interaction with MOM. It determines regions where traffic is expected to be unacceptably dense or difficult to manage (complex), and resolves such problems by maneuvering traffic. A key factor in determining what traffic is too dense or too complex is whether MOM is able to handle it.

C.2 MOM AS AN INTERMEDIATE AERA 3 SHELL

MOM is planned as the next inner shell in AERA 3. MOM generally looks five to thirty minutes ahead, when possible separation problems between pairs of aircraft can be profitably detected, analyzed and placed in a global context, but when prediction uncertainties preclude either

(a) determination of which possible pairwise separation problems will actually require aircraft to be maneuvered, and (b) routine determination of the best resolutions for specific possible pairwise separation problems.

C.2.1 Possible Pairwise Separation Problems (Possiblems)

Possible pairwise separation problems are a key idea in MOM analysis. For convenience (and to emphasize via a coined word the fact that the idea is not used in ATC now), the four-word phrase is abbreviated "possiblem".

Figure C-1a illustrates how possiblems might be detected. A box is placed around each aircraft, large enough to reflect the predicted position uncertainties of the aircraft, as well as a separation buffer. The boxes move with time. A possiblem occurs anytime the boxes for two aircraft are predicted to bump, since the aircraft themselves may then come close enough to require a separation maneuver.

C.2.2 Clusters

A set of possiblems involving a set of aircraft is defined as a cluster, another key MOM idea. For example, possiblems between Aircraft A and B, and between B and C, are said to form a three-aircraft, two-possiblem cluster. A separate cluster might involve four other aircraft (say, D, E, F, and G), with five possiblems (say, D-E, D-F, D-G, E-F, and F-G). Clusters can be illustrated graphically, as in Figure C-2a, where each aircraft is represented by a node, and each possiblem by an edge between two nodes.

C.2.3 Available Maneuver Options (Outs)

MOM determines which of several simple maneuver options is available (free of possiblems) for each aircraft. These options involve limited displacements left/right/ahead/behind/above/below nominal. The displacements are modeled to begin not immediately, but typically some minutes in the future, usually just a few minutes prior to the possiblems.

Figure C-1B illustrates how constraints on maneuver options might be detected. It is similar to Figure C-1a, except that regions are added left, right, ahead, behind, above, and below Aircraft A's box; if any of these regions bump B's box, Aircraft B constrains the respective maneuver option for Aircraft A.

An available maneuver option not only eliminates all of an aircraft's possiblems; it also causes no new ones(assuming the other aircraft maintain their trajectories within a tolerance).

C.2.4 Simplification of Clusters

MOM simplifies a cluster by protecting (for future use) an out for one or more of the involved aircraft. The future use of the out causes a cluster to be broken into independent, smaller, and less complex clusters.

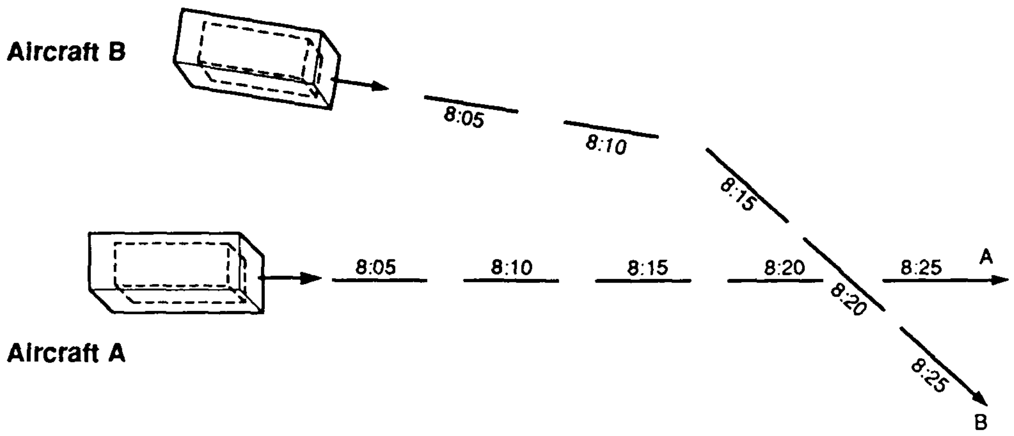

FIGURE C-1a
A POSSIBLE PAIRWISE SEPARATION PROBLEM (POSSIBLEM) EXISTS IF BUFFERED BOXES ABOUT EACH AIRCRAFT, MOVING IN TIME, EVER BUMP

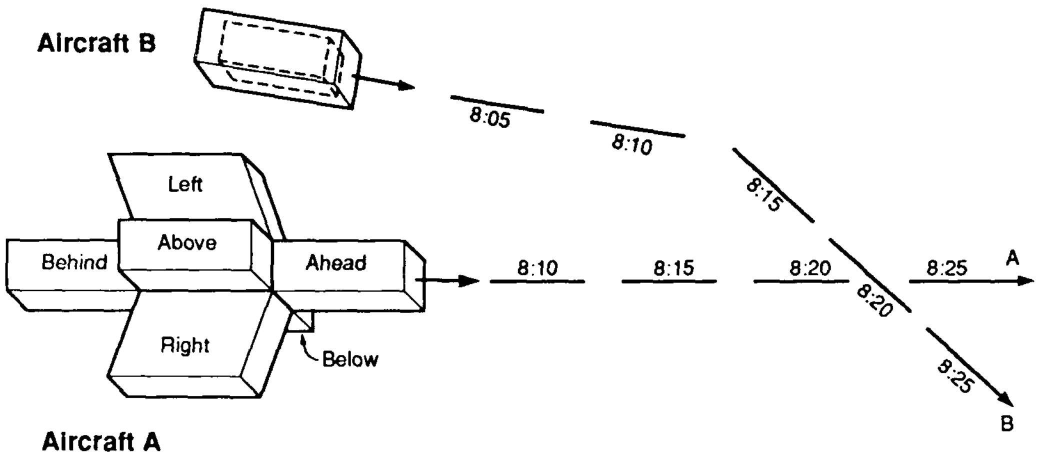

FIGURE C-1b
A's VARIOUS MANEUVER OPTIONS ARE CONSTRAINED BY B IF THE CORRESPONDING REGIONS ATTACHED TO A's BOX EVER BUMP B's BOX

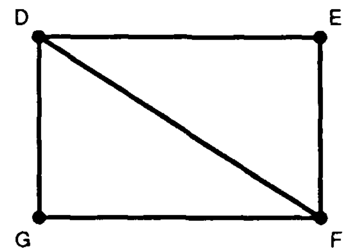

FIGURE C-2a
A FOUR-AIRCRAFT (NODE), FIVE-POSSIBLEM (EDGE) CLUSTER

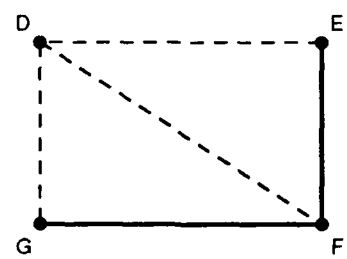

FIGURE C-2b
IF D MANEUVERS VIA A MOM OUT, ONLY THE 2 POSSIBLEMS NOT INVOLVING D REMAIN

For instance, an out for Aircraft D in the above example would eliminate three of the cluster's five possiblems (the three involving D), thereby reducing the cluster to two possiblems (both involving F). A second out (for F) eliminates the remaining two possiblems. No new possiblems are created versus any other aircraft (however, a separate check must be made that D's and F's outs are compatible with each other and do not create a D-F possiblem).

MOM maintains at all times a contingency plan, consisting of a set of outs that simplify all clusters to a level necessary to assure that AERA 3's inner shell can resolve any remaining separation problems. This level may consist of isolated possiblems whose aircraft have outs.

Occasionally, MOM identifies, or designates, a particular out as providing enough planning benefits that AERA 3 should commit early to it (to some degree), despite the fact that more accurate information will become available in the future. An active area of research is to determine which outs to designate. Attractive candidates are outs which resolve many possiblems at once, especially possiblems with high certainty of requiring a separation maneuver.

C.3 INNER SHELL IN AERA 3

The innermost AERA 3 shell is the Automated Separation Function (ASF). ASF is loosely analogous to today's ATC at the sector level. ASF has several parts, but its main function is to determine detailed resolutions for specific pairwise separation problems (aircraft versus aircraft or airspace). It seeks resolutions which are safe and which impose minimal penalty upon the aircraft (time, fuel, etc.). Its lookahead is less than ten minutes, over which prediction uncertainties are relatively small. It uses track-based data as well as trajectory data (whereas all outer shells use primarily trajectory data).

ASF generally considers one pairwise separation problems one (or a very few) at a time, an approach that is justified from a systems planning point of view given that MOM breaks down clusters involving multiple interrelated possiblems. ASF relies upon input from MOM to recommend certain outs, and also to prohibit certain outs with adverse downstream effects on other traffic.

C.4 AERA 3 NESTED SHELLS IN CONTEXT

The hierarchy of nested shells in AERA 3 works as follows:

- ASF separates aircraft;

- MOM assures that ASF can operate successfully in a global context (though ASF considers pairwise separation problems one at a time);

- The Airspace Manager Planning Functions (particularly Density/Complexity Manager) assures, among other things, that MOM can operate successfully by preventing traffic densities that cause aircraft to have no outs.

APPENDIX D

SYMBIOSIS BETWEEN MOM AND GS

This section explores the symbiosis between MOM[5] and GS, first mentioned in Section 1. An overview of MOM is presented in Appendix C.

The usefulness of GS to MOM lies in the fact that GS's mathematical relationships between the geometry (a), the separation achieved (b), the uncertainties (c), and the gentleness (d) are calculable offline, and are therefore available for MOM to apply to clusters tens of minutes in the future. More specifically, GS provides the following benefits to MOM:

- GS resolutions are bounded in how far aircraft are displaced from nominal.
- Knowing that a GS resolution is available for certain pairwise possible separation problems (possiblems), MOM can determine that safe separation is guaranteed even if there are no outs.
- Knowing that a GS resolution is available for each possiblem in certain clusters involving several possiblems, MOM can determine that safe separation is guaranteed, with no designated outs. More generally, if GS resolutions are available for some possiblems, MOM can determine that safe separation is guaranteed, with fewer designated outs than otherwise necessary. Figure D-1 summarizes the symbiosis. MOM helps to simplify clusters of interrelated possiblems into isolated possiblems. GS then assures that gentle maneuvers suffice for the isolated possiblems. Since the maneuvers are gentle, the downstream impact of GS maneuvers is minimal. This helps maintain the integrity of MOM's database, which MOM depends upon to continue to simplify clusters as time passes—the cycle is complete.

D.1 EXAMPLE APPLICATION OF GS BY MOM: SINGLE POSSIBLEM WITH NO OUTS

For certain possiblems 20 or 30 minutes in the future, the prediction uncertainties may be so large that MOM finds no specific left or right outs for either aircraft—because left and right maneuver options are constrained by the other aircraft (and, let us suppose, by that aircraft only). Assume also that the other four maneuver options (above, below, ahead, behind) are constrained (by additional aircraft, performance limitations, etc.). In this case, MOM finds no outs for either aircraft. However, MOM can apply the GS mathematical relationships to assure that a safe resolution exists (and a gentle one at that). It may not be clear (at MOM's lookahead) which aircraft needs to maneuver, or whether the maneuver will be right or left, but the key point is that some gentle, safe out will become available.

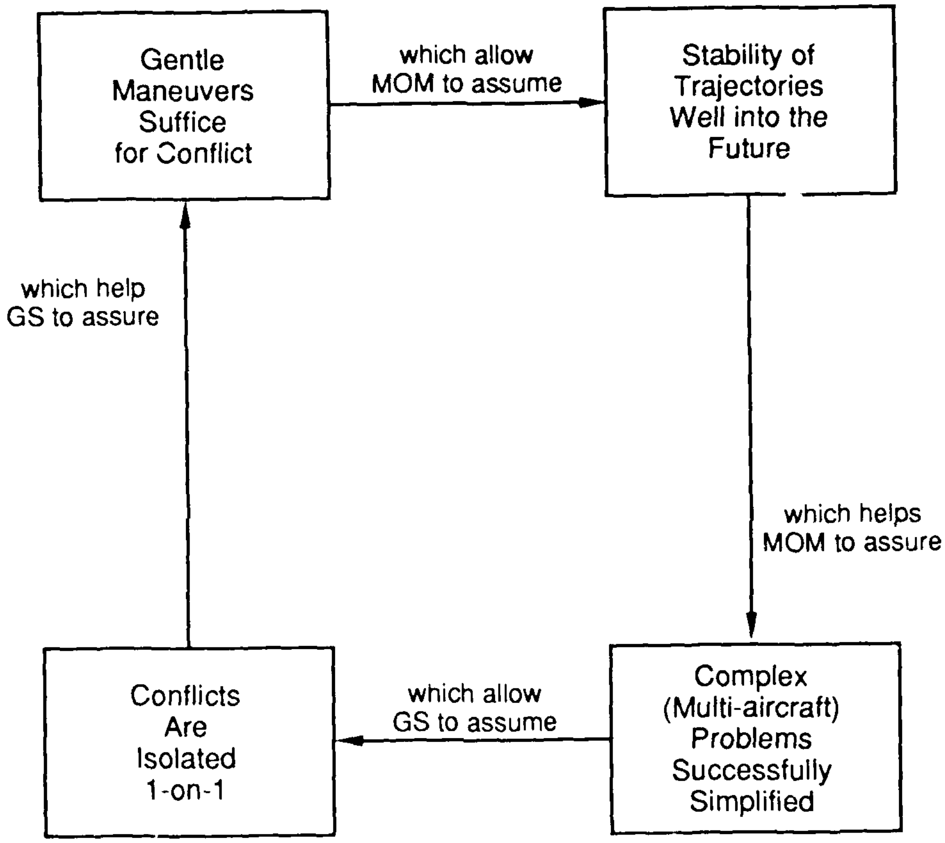

FIGURE D-1
SYMBIOSIS BETWEEN MOM AND GS

D.2 EXAMPLE APPLICATION OF GS BY MOM: SEQUENTIAL POSSIBLEMS WITH NO OUTS

Another useful feature of GS for MOM is that GS can be applied iteratively, for sets of possiblems sufficiently separated in time. This feature is a result of GS's consideration only of parallel lateral offset maneuvers; these offsets, once attained (in just 2-4 minutes), restore the geometry as parameterized in (a); the encounter angle and the relative speed are unchanged.

For example, suppose Aircraft X has three possiblems (versus Aircraft A, B, and C) which occur, respectively, 10, 20, and 30 minutes in the outs. However, suppose that X's left and right maneuver options are constrained only by Aircraft A, B, and C, and also that the left and right maneuver options of A, B, and C are constrained only by X. The ahead, behind, above and below maneuver options for all four aircraft may well be constrained by additional aircraft. Then, 10, 20, and 30 minutes hence, MOM is assured that:

- Possiblem X-A can be resolved by a GS gentle parallel offset (by X or A).
- Possiblem X-B can be resolved by a GS gentle parallel offset (by X or B), regardless of whether or not X was maneuvered for the X-A possiblem.
- Possiblem X-C can be resolved by a GS gentle parallel offset (by X or C), regardless of whether or not X was maneuvered for the X-A or S-B possiblems.

Despite the fact that none of the four aircraft have a specific out, MOM knows, thanks to GS, that sufficient outs will become available as needed, for these three possiblems.

D.3 EXAMPLE APPLICATION OF GS BY MOM: SIMULTANEOUS POSSIBLEMS WITH NO OUTS

Consider a "chain" of possiblems A-B, B-C, C-D, D-E, E-F, etc., which occur (respectively) 10, 20, 10, 20, 10, 20, etc., minutes in the future (Figure D-2b). As before, suppose that each aircraft has zero outs, but that each aircraft's left and right maneuver options are constrained only by aircraft having possiblems with that aircraft (e.g., D is constrained leftward/rightward only by C and E). The later (20-minute hence) possiblems can be resolved by GS maneuvers, independently of which aircraft are maneuvered via GS resolutions for the early (10-minute hence) possiblems.

For an arbitrarily long chain of possiblems of this sort, MOM again knows, thanks to GS, that sufficient outs will become available, although none of the aircraft has a specific out at the current time.

D.4 ADDITIONAL EXAMPLES

More complex examples can also be constructed, combining elements of the two examples illustrated in Figures D-2a and D-2b.

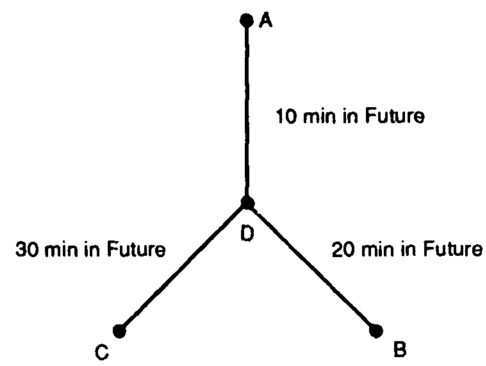

FIGURE D-2a
THREE SEQUENTIAL POSSIBLEMS

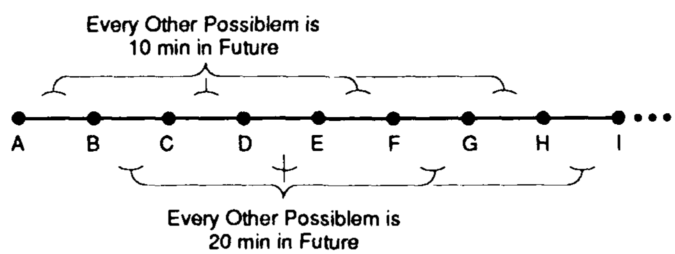

FIGURE D-2b
MULTIPLE POSSIBLEMS 10-20 MINUTES IN FUTURE

APPENDIX E

PARAMETERIZING GENTLENESS AND TIMELINESS FOR GS MANEUVERS

In this appendix, various issues are discussed involving the parameterization of gentleness and timeliness in GS maneuvers. These issues are discussed briefly in Section 2 and in more detail here.

In Section 2.5, gentleness is parameterized via the magnitude of the parallel lateral offset (u) and the delay induced (v). In some applications of GS, upper bounds on these values are specified a priori; these upper bounds are denoted uHI and vHI.

In Section 2.10, timeliness of a GS maneuver is parameterized. The motivation is to set limits on how early a parallel lateral offset maneuver must begin, in order that the GS mathematical relationships apply. Timeliness parameters, which include a lower bound, vLO, on the induced delay v, are introduced to quantize the intuitive notion that if a lateral offset takes only a few minutes to achieve, it need start only a few minutes prior to the conflict.

E.1 DISCUSSION OF PARAMETERIZATION OF GENTLENESS

Gentleness is parameterized via the following limits:

1. An upper bound on the lateral offset u, denoted uHI, to keep the aircraft laterally close to its nominal path.

2. An upper bound on the induced longitudinal delay v, denoted vHI, to assure the maneuvered aircraft is delayed by only a few seconds

Limit 1 is based on the general assumption that most users will be on their preferred trajectories, or on the best alternate route the ATC system can provide. The maximum distance an aircraft is displaced from its current (read, preferred) trajectory is therefore a plausible measure for how gentle the maneuver is.

Limit 2 is perhaps the most obvious measure of a maneuver's gentleness. Delay not only causes late arrivals, but also imposes a fuel penalty. It also degrades the effectiveness of the long lookahead functions which take (will have been taking) actions based upon the original trajectories.

As a general rule, the ability of a maneuver to provide separation in a given geometry is inversely proportional to the maneuver's gentleness, as measured by (1) and/or (2).

To some extent, the limits (1) and (2) are linked. For instance, if the parallel lateral offset is implemented as a turn of ψ degrees away from nominal and (once the full offset u is reached) of ψ degrees back again (which closely approximates a realistic ATC maneuver, in which u would be at least several times larger than v), then u grows linearly with v (for constant ψ).

E.1.1 Default Value for uHI

The GS analysis assumes that the two aircraft in conflict are free of interactions with third aircraft in the vicinity of their conflict; limit 1 allows quantization of "vicinity". This quantization helps not only to make the assumptions underlying GS itself more exact, but also to place the MOM-GS symbiosis on a firmer foundation.

A reasonable value for uHI with which to present numerical results here is 12 nmi. A lateral maneuver of this magnitude, assuming several minutes to achieve the full offset, qualifies as gentle by the standards of today's controllers and pilots. It also proves effective (from the figures in Section 3). The value appears as well to be in the right ballpark, at least, for the definition of "vicinity" and for use in the MOM symbiosis (although the supporting evidence from MOM analysis is admittedly high preliminary).

Note that the definition of "vicinity" would have to include a buffer (of a bit more than 5 nmi, perhaps) beyond uHI to the left/right, so that a GS maneuver cannot cause a conflict with a third aircraft.

E.1.2 Default Value for vHI

Ideally, the induced delay is made as small as possible, since its magnitude helps determine time/fuel cost to the user, as well as the longitudinal uncertainty modeled by MOM and the other long lookahead AERA 3 functions. The smaller the delay, the less uncertainty is imposed upon those functions, and the denser the traffic that can be handled.

A value of vHI = 2 nmi appears to be reasonable for presentation of Section 3's numerical results. This value indeed keeps the induced delay time reasonably small, and yet allows the GS maneuvers to be effective. For aircraft at the following speeds (in knots), the maximum induced delay (in seconds) is given:

SPEED	INDUCED DELAY (vHI = 2 nmi)
480 kts	15.0 seconds
420 kts	17.1 seconds
360 kts	20.0 seconds

Note that (2) measures delay only in a localized sense. Should the maneuvered aircraft's original trajectory turn rightward 10 minutes after a rightward GS maneuver, the ultimate delay is overestimated; if the original trajectory turns leftward 10 minutes later, the ultimate delay is underestimated.

E.2 DISCUSSION OF TIMELINESS PARAMETERS

It is desirable for resolution maneuvers to start only a few minutes prior to the conflict. The main reason for this is that the uncertainty parameters (particularly the longitudinal ones Lons and Lonf) grow as a function of the time between the beginning of the maneuver and the conflict. Secondly, for GS to consider long lookahead resolutions would complicate analysis fo the symbiosis between GS and MOM (and perhaps compromise the symbiosis itself). Thirdly, lateral resolutions that start too early simply make no ATC sense. For example, a 10-mile lateral offset maneuver that begins 20 minutes prior to the conflict (e.g., a 5-degree turn at H) would have a very small induced delay v, and might appear attractive (looking only at u and v), but would be inappropriate for AERA 3.

E.2.1 First Close Approach

To quantify the notion that a GS maneuver ought not to start too early, a limit is placed on distance HQ, where Q, called the point of first close approach, is defined as:

- the point at which the aircraft to be maneuvered first comes within a parameter distance of the other aircraft.

In this document, the distance is always assumed to be 12 nmi. H, as before (see Figure E-1), is the point where the maneuver begins. Depending on the relative timing T, and the maneuver selected, Q could come either before or after point J and before or after point P in Figure E-1.

"First close approach" is used instead of "closest approach" (which might seem more natural), because in many geometries, the aircraft are close together for a long time prior to closest approach, and a GS maneuver should not be "penalized" for beginning (as it must) well prior to closest approach.

It is convenient to express the distance HQ as HJ + JQ, where JQ is considered negative if point Q occurs prior to point J along the aircraft's trajectory. Thus, timeliness is quantized via the sum of two parameters:

(a) A bound on the interval from H to J

(b) A bound on the interval from J to Q, the point of first approach

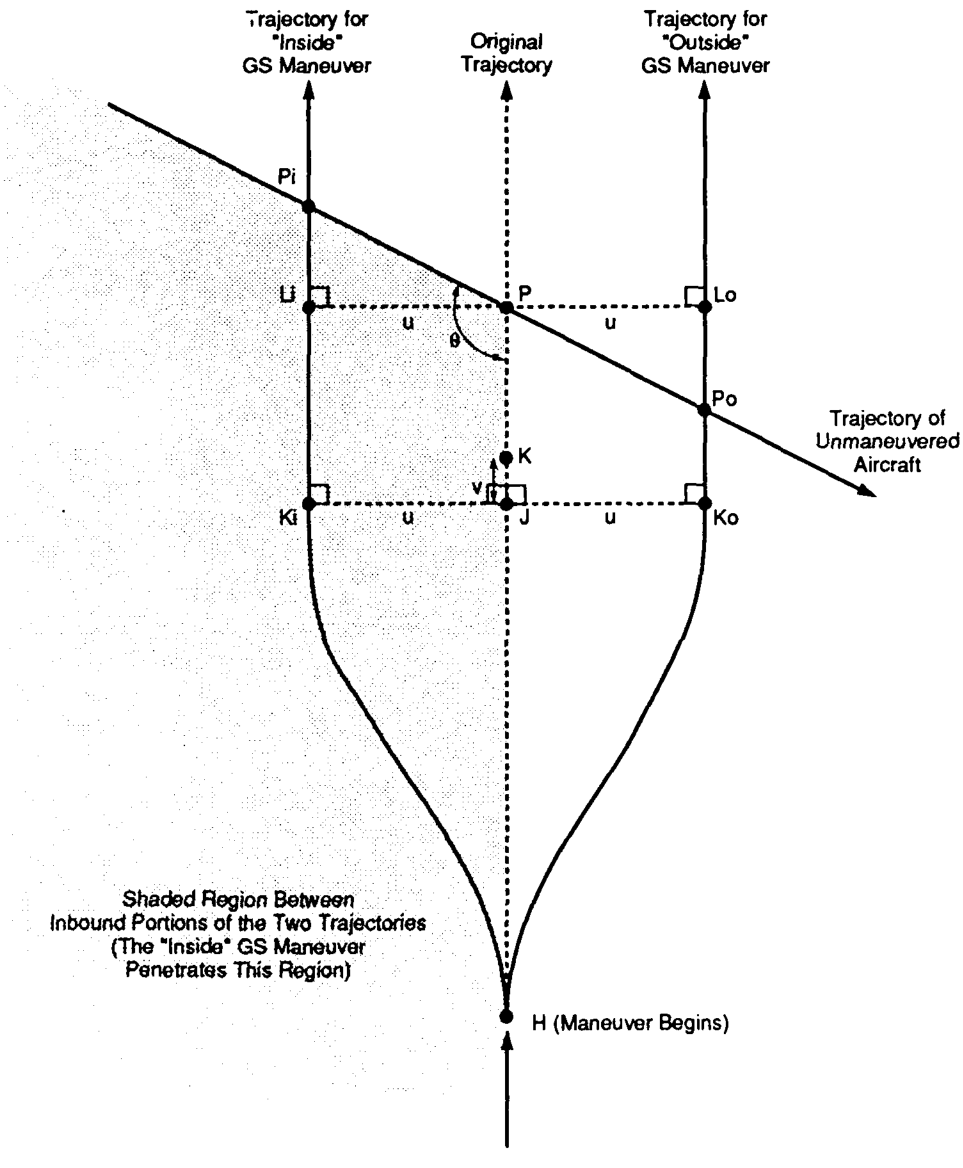

FIGURE E-1
A TYPICAL GS ENCOUNTER GEOMETRY

E.2.1.1 Bound on HJ

HJ represents the time to achieve the offset u (starting at zero offset). The only advantage for GS in specifying a maneuver with a large interval from H to J is to allow the maneuver toe extremely gentle so that the induced delay, v, is extremely small. For a given lateral offset u, the delay v can be made arbitrarily small, but only if the interval from H to J is made unrealistically long (e.g., tens of minutes), in which case the maneuver is generally inappropriate for pairwise conflict resolution.

Maneuvers that need start only a short time before the conflict, on the other hand, minimize uncertainties and unnecessary impact upon users.

Reasonable values for HJ are about 4 minutes or 30 nmi for a 12 nmi lateral offset.

The limit on the interval from H to J can be accomplished effectively (though indirectly) via vLO, a lower bound on v. This indirect bound proves useful in simplifying the analysis to come. The lower bound vLO renders GS maneuvers with unacceptably long HJ physically impossible.

A reasonable value for vLO is 2 nmi (i.e., equal to the default value for vHI). Using vLO = vHI, GS can achieve the minimum separation promised by GS_SEP by using maneuvers that are congruent (regardless of which aircraft is moved and in which direction). Use of this value implies that the 12 nmi offset is achieved over a longitudinal distance of about 30 nmi and therefore takes about 4 minutes.

It can be proven that the separation achievable via the GA maneuver FO is dependent upon vLO but not on vHI; whereas the separation achievable via the other three GS maneuvers (SI, SO, FI) is dependent upon vHI but not on vLO. The separation achieved via SI, SO, and FI grows as v gets larger (and all other parameters are held constant), but the separation achieved via FO grows as v gets smaller, for all values of r and θ.

E.2.1.2 Bound on JQ

The limit on JQ, unlike that on HJ, is difficult to specify in the GS mathematical formulations. Instead, an upper bound is found for JQ (see Appendix F, Sections 11 and 12). This upper bound confirms in a quantitative fashion the intuitive notion that if the lateral offset takes only a few minutes to achieve, it need start only a few minutes before the conflict.

APPENDIX F

DETAILED GS MATHEMATICAL DERIVATIONS

In this appendix, the basic mathematical formulas expressing GS performance are derived. A glossary of key terms and mathematical symbols appearing in this appendix is given in Table F-1. The key equations are summarized in Table F-2.

F.1 FINDING DISTANCE BETWEEN AIRCRAFT AS A FUNCTION OF TIME

Figure F-1 shows a crossing conflict between Aircraft A and B, whose:

- Respective speeds are S_a and S_b
- Trajectory intersection is at point P
- Respective nominal times to reach the intersection P are T_a and T_b
- Encounter angle is θ (angle between the rays approaching P)

For the moment, assume the aircraft follow their trajectories exactly. (Errors in pathkeeping will be incorporated into the derivation in Section F.6.) Without loss of generality, assume A is the faster aircraft.

By the law of cosines, the square of the distance D(t) between A and B at any time t is:

(1) $D^2(t) = (t - T_a)^2 S_a^2 + (t - T_b)^2 S_b^2 - 2S_aS_b (t - T_a)(t - T_b)\cos\theta$

In the figure, B (the slower aircraft) has reached the intersection but A has not (i.e., $(t - T_a)$ and $(t - T_b)$ have opposite signs); note that $(-\cos\theta)$ has been substituted for $\cos(180 - \theta)$. Equation (1) is a general formulation, however, and applies for all t and all $0<\theta<180$.

It will be convenient to let zero along the time (t) axis be fixed at T_a, and to define:

(2)	$T = T_b - T_a = T_b$	(T represents the lead in minutes the FASTER aircraft has over the SLOWER; if T<0 the SLOWER reaches the intersection ahead of the FASTER)
(3)	$r = S_b / S_a$	(speed ratio of slower to faster aircraft; $0<r\leq1$)

From now on, to keep the notation simpler, the symbol s will be used to denote the FASTER aircraft's (A's) speed, while rs will denote the SLOWER aircraft's speed. The lower case "s" is used for "speed", reserving the upper case "S" for "SLOWER".

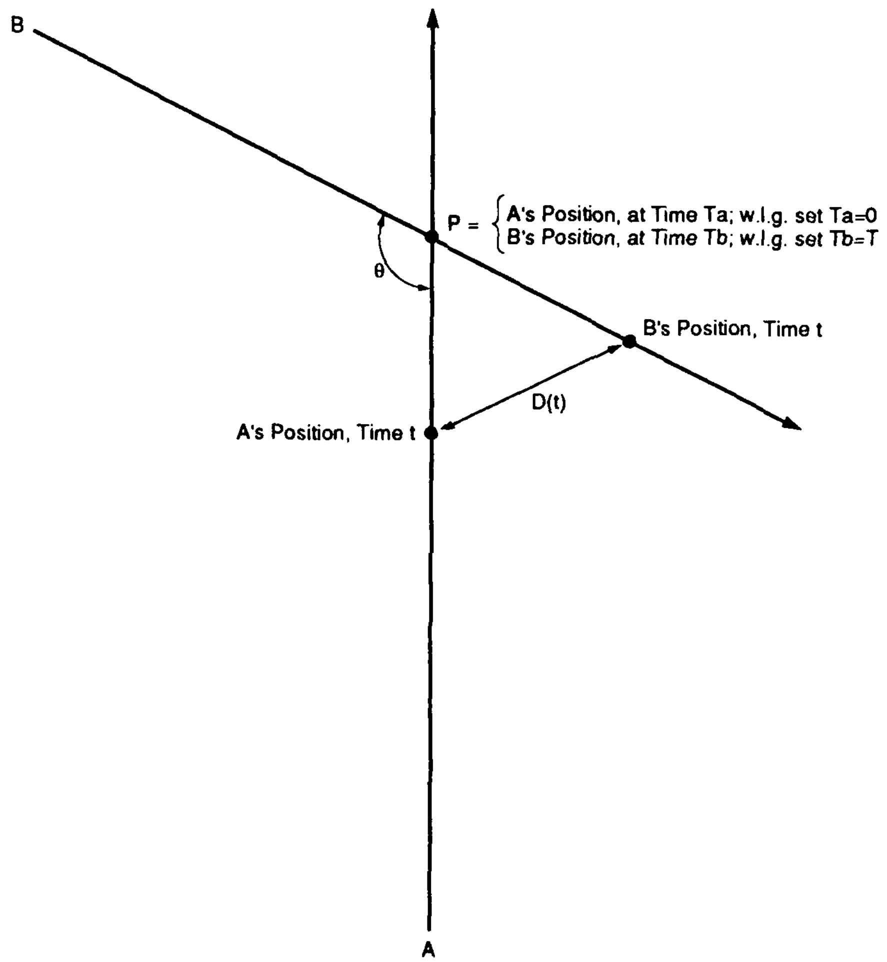

FIGURE F-1
TIMING PARAMETERS FOR A GS GEOMETRY

TABLE F-1

GLOSSARY OF KEY TERMS AND SYMBOLS USED IN APPENDIX F

D	The minimum separation between two aircraft, assuming no GS maneuver
D'	The minimum separation between two aircraft, after a GS maneuver
D(t)	The distance between two aircraft at time T (assuming they follow their trajectories exactly
DELTA(T)	The change in relative times to intersection as a result of a GS maneuver
EFFECTIVE_TAU	A measure for how early a GS maneuver must begin
Faster Inside	A GS maneuver whereby the faster aircraft maneuvers toward the inbound portion of the slower aircraft's trajectory
Faster Outside	A GS maneuver whereby the faster aircraft maneuvers away from the inbound portion of the slower aircraft's trajectory
FI	Faster Inside
FO	Faster Outside
Gentle	Describes a parallel lateral offset maneuver whose displacement from nominal is bounded by parameters
GFA	Get Faster Ahead; the strategy of achieving safe separation by having the faster aircraft reach the intersection first.
GS	Gentle Strict
GSA	Get Slower Ahead; the strategy of achieving safe separation by having the slower aircraft reach the intersection first.
GS_SEP	The amount of separation guaranteed achievable by a GS maneuver, for a given geometry and for given bounds on gentleness
Inside Maneuver	FI or SI
Latf	Lateral uncertainty for the faster aircraft

TABLE F-1

GLOSSARY OF KEY TERMS AND SYMBOLS USED IN APPENDIX F (Continued)

Lats	Lateral uncertainty for the slower aircraft
Lonf	Longitudinal uncertainty for the faster aircraft
Lons	Longitudinal uncertainty for the slower aircraft
MANEUVER_TIME	The amount of time needed to achieve a given parallel offset
NEEDED_SEP	The desired amount of separation between aircraft
Outside Maneuver	SO or FO
PRE_TIME	The time elapsed from the time GS must commit to a maneuver, until the (unmaneuvered) aircraft first become close
r	The ratio of the slower aircraft's (ground) speed to that of the faster
s	The faster aircraft's ground speed
SI	Slower Inside
Slower Inside	A GS maneuver whereby the slower aircraft maneuvers toward the inbound portion of the faster aircraft's trajectory
Slower Outside	A GS maneuver whereby the slower aircraft maneuvers away from the inbound portion of the faster aircraft's trajectory
SO	Slower Outside
T	The difference between the time the slower aircraft reaches the intersection and the time the faster aircraft does so (assuming no GS maneuver)
T'	The value of T after a GS maneuver; T' = T + DELTA(T)
TAU, TAU_x, TAU_y	Measures for how late a GS maneuver may begin
T_BREAKEVEN	The value of T for which GS strategies GSA and GFA do equally well, and for which GS provides the least guaranteed separation

TABLE F-1

GLOSSARY OF KEY TERMS AND SYMBOLS USED IN APPENDIX F (Concluded)

THETA	The encounter angle (θ)
t(minsep)	Time of minimum separation between two aircraft
u	The magnitude of a GS parallel offset
uHI	The largest parallel offset qualifying as "gentle"
v	Symbol for delay (in nmi) induced by a GS maneuver
vLO	Lower limit on induced delay, v
vHI	Upper limit on induced delay, v
W	A symbol for the quantity ($r\ s\ \sin\theta$)
Y	A symbol for the quantity SQRT($r^2 - 2r\cos\theta + 1$)
X	A term expressing the effect upon GS_SEP of longitudinal and lateral uncertainties

TABLE F-2

THE GENTLE-STRICT MATHEMATICAL DERIVATIONS: OVERVIEW

I DECIDE WHICH OF THE FOUR GS MANEUVERS (FO< FI< SO< SI) IS BEST, FOR TWO STRATEGIES

A. For Getting Slower Aircraft Ahead—Use FI (faster inside)

B. For Getting Faster Aircraft Ahead, consider three inequalities:

(1) $\cos\theta > r$

(2) $uHI\ (1 + r)\ (1 - \cos\theta) > (vHI + (r\ vLO))\ \sin\theta$

(3) $uHI\ (1 - r)\ (1 + \cos\theta) > (vHI + (r\ vLO))\ \sin\theta$

IF (2) and (3) then—Use FO (faster outside)	Call this "FO vs. FI"
Else if (1) then—Use SO (slower outside)	Call this "SO vs FI"
Else—Use SI (slower inside)	Call this "SI vs. FI"

II DETERMINE WHICH STRATEGY TO USE

Compare "GET FASTER AHEAD" (via SO, SI, FO) with "GET SLOWER AHEAD" (via FI)
(Decision Depends on T, So Cannot Be Made until Just Before Conflict)

SO vs. FI:	Use SO	rather than FI if $T < [uHI(1+r)(1-\cos\theta)\quad (1-r)\sin\theta\ (vHI)]$	$/ (2r\ s\ \sin\theta)$
SI vs. FI:	Use SI	rather than FI if $T < [uHI(1-r)(1-\cos\theta) - (1-r)\sin\theta\ (vHI)]$	$/ (2r\ s\ \sin\theta)$
FO vs. FI:	Use FO	rather than FI if $T < [\ +(r)\ \sin\theta\ (vHI+vLO)]$	$/ (2r\ s\ \sin\theta)$

III DETERMINE GS_SEP, THE GUARANTEED MINIMUM SEPARATION ACHIEVABLE

(Does not depend on T, so can be determined long before conflict; does not depend on s, either)

SO vs. FI: $GS\_SEP \geq [uHI(1+r)(1-\cos\theta) + (1+r)\sin\theta(vHI)\quad -2X] / [2\ SQRT(r^2-2r\cos\theta + 1)]$

SI vs. FI: $GS\_SEP \geq [uHI(1+r)(1-\cos\theta) + (1+r)\sin\theta(vHI)\quad - 2X] / [2\ SQRT(r^2-2r\cos\theta + 1)]$

FO vs. FI: $GS\_SEP \geq [2uHI(1-r\cos\theta) + (r)\sin\theta\ (vHI-vLO)\quad - 2X] / [2\ SQRT(r^2-2r\cos\theta + 1)]$

TABLE F-2

THE GENTLE-STRICT MATHEMATICAL DERIVATIONS: OVERVIEW (Concluded)

NOTATION: VARIABLES ACCURATELY KNOWN LONG BEFORE THE CONFLICT

uHI (nmi)	Maximum lateral offset allowed
vHI (nmi)	Maximum (rearward) longitudinal offset allowed (floor on user's delay per conflict; supports MOM symbiosis)
vLO (nmi)	Minimum (rearward) longitudinal offset allowed (cannot get lat offset u reasonably quickly without delay $\geq$ vLO
θ	Encounter angle "THETA"
r	Ratio of speed of slower to speed of faster aircraft
s	Speed of faster aircraft
Lonf, Lons (nmi)	$\pm$ Longitudinal uncertainty (for faster, slower aircraft, respectively)
Latf, Lats (nmi)	$\pm$ Lateral uncertainty (for faster, slower aircraft, respectively)

ADJUSTMENT FOR LATERAL AND LONGITUDINAL UNCERTAINTIES:

X $\quad$ (Lons + (r Lonf) $\sin\theta$ + Lats [ABS($\cos\theta$ - r)] + Latf (1 - r $\cos\theta$)]

VARIABLE KNOWN ACCURATELY ONLY SHORTLY BEFORE THE CONFLICT

T $\quad$ (Slower aircraft's time to intersection) - (Faster aircraft's time)

Substituting (2) and (3) into (1) yields:

$$(4) \quad D^2(t) = s^2 \, [t^2 + r^2 (t - T)^2 - 2rt \, (t - T) \cos\theta]$$

F.2 FINDING TIME AT WHICH DISTANCE IS MINIMIZED

To determine the time t(MINSEP) at which the separation between the two aircraft is minimized, the derivative of (4) is set to zero:

$$(5) \quad \frac{d[D^2(t)]}{dt} = 0 = s^2 \, [2t + 2r^2(t - T) + (2rT - 4rt) \cos\theta]$$

Solving for t = t(MINSEP) yields:

$$(6) \quad t(\text{MINSEP}) = \frac{Tr(r - \cos\theta)}{r^2 - 2r \cos\theta + 1}$$

F.3 FINDING THE MINIMUM SEPARATION BETWEEN THE TWO AIRCRAFT

Substituting (6) into (4) (and taking the square root) yields (7), an expression for the minimum separation between the two aircraft, denoted simply as D. This is a long derivation-a useful first step is that, from (6):

$$[t(\text{MINSEP}) - T] = T(r \cos\theta - 1) \, / \, (r^2 - 2r \cos\theta + 1)$$

$$(7) \quad D = D[t(\text{MINSEP})] = \frac{\text{ABS}(T) \, r \, s \sin\theta}{\text{SQRT}(r^2 - 2r \cos\theta + 1)}$$

F.4 FINDING MINIMUM DISTANCE AFTER A GS MANEUVER

Now consider Figure F-2 (identical to Figure 2-3). It shows the two possible GS maneuvers for A. These two maneuvers are FASTER INSIDE, or FI, achieved via H-K_i-L_i-P_i (into the angle θ and into the shaded region); and FASTER OUTSIDE, or FO, achieved via H-K_o-P_o-L_o (away from the angle θ and outside the shaded region).

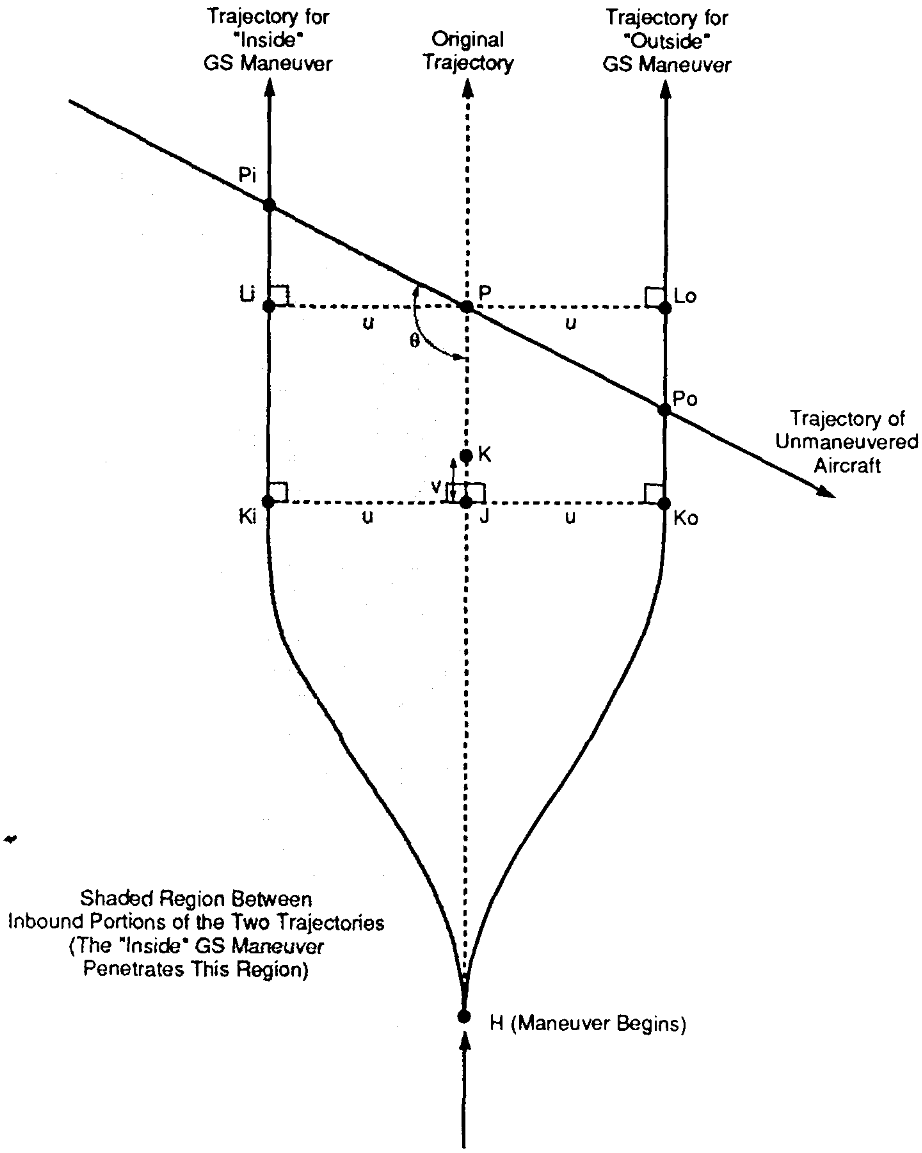

FIGURE F-2
A TYPICAL GS ENCOUNTER GEOMETRY

To keep the analysis tractable, it is assumed that aircraft A, via FI (or FO), reaches K_i (or K_o), and its full parallel offset u nmi, prior to the (new) time of minimum separation between A and B. (Later, in Section F.11, the impact of this assumption will be explored in detail.)

Equation (7) can be used to determine the new minimum separation, denoted D', as a result of a GS maneuver (FI or FO). Note that the values for r (the speed ratio), s (A's speed), and θ (the encounter angle) are unchanged by the FI or FO maneuver. Only T, the difference between B's and A's arrival times at the new intersection (P_i or P_o), is changed (from its value for the old intersection, P). Let the new value be denoted T'. Then (swapping T' for T in (7)), the new minimum separation is:

$$(8) \quad D' = \frac{ABS(T')\, r\, s\, \sin\theta}{SQRT(r^2 - 2r\cos\theta + 1)}$$

If T and T' have the same sign, the order in which A and B reach the intersection of their trajectories is unchanged by the parallel offset (positive T means A, the faster aircraft, arrives first; negative T means B arrives first). If, in addition, $|T'| > |T|$, then whichever aircraft originally was ahead increases its lead as a result of the parallel offset, and the minimum separation D' increases (from D) by the amount:

$$(9) \quad DELTA(D) = \frac{ABS\,(DELTA(T))\, r\, s\, \sin\theta}{SQRT(r^2 - 2r\cos\theta + 1)}$$

where DELTA(T) is the difference between the new and old differences between B's and A's arrival times at the intersection of their trajectories:

$$\begin{aligned}(10) \quad DELTA(T) = T' - T &= (\text{new } T_b - \text{new } T_a) - (\text{old } T_b - \text{old } T_a)\\ &= (\text{new } T_b - \text{old } T_b) - (\text{new } T_a - \text{old } T_a)\end{aligned}$$

If $|T'| < |T|$ (and T and T', as before, have the same sign), DELTA(D) is instead decreased by the expression given in (9).

F.5 DETERMINING DELTA(T) FOR ALL FOUR GS MANEUVERS

Figure F-2 is now used to illustrate how to compute DELTA(T) for parallel offsets of u nmi, with induced longitudinal delays of v nmi, via a GS maneuver.

Thanks to the INSIDE/OUTSIDE notation (rather than LEFT/RIGHT, as discussed in Section 2.2), a conflict with an aircraft coming in from the left at angle θ (as shown in Figure F-2) can be treated in exactly the same way as the mirror image geometry, where the aircraft comes in from the right at angle θ. θ therefore need range only between 0 and 180 degrees; negative θ need not be considered.

For an INSIDE turn, A's path to the new intersection P_i is changed (relative to its path to P) by:

$(JK + P_iL_i) = (v - u/\tan\theta)$ nmi, or $((v - u/\tan\theta) / s)$ min.

(Note that $\tan\theta < 0$ in the figure). B's path to the new intersection P_i is changed by :

$-(P_iP) = -u/\sin\theta$ nmi, or $((-u/\sin\theta) / rs)$ min.

Thus, for A (the FASTER aircraft) INSIDE:

DELTA(T) = $(\text{new } T_b - \text{old } T_b) - (\text{new } T_a - \text{old } T_a) = -(u / (r\,s \sin\theta)) - ((v - u / \tan\theta) / s)$

DELTA(T) for FASTER OUTSIDE can be computed in a similar way. One can also use Figure F-2 to compute DELTA(T) for the SLOWER aircraft's (B's) maneuvers, simply by letting the northbound aircraft represent B instead of A. The result is:

				(new time) - (old time) to intersection for slower aircraft		(new time) - (old time) to intersection for faster aircraft
(11a)	DELTA(T)	(SO)	=	$[+u/(rs\tan\theta) + v/(rs)]$	-	$[u/(s\sin\theta)]$
(11b)	DELTA(T)	(SI)	=	$[-u/(rs\tan\theta) + v/(rs)]$	-	$[-u/(s\sin\theta)]$
(11c)	DELTA(T)	(FO)	=	$[+u/(rs\sin\theta)]$	-	$[u/(s\tan\theta) + v/(s)]$
(11d)	DELTA(T)	(FI)	=	$[-u/(rs\sin\theta)]$	-	$[-u/(s\tan\theta) + v/(s)]$

Using as a common denominator $(rs\sin\theta)$, the above are expressed more conveniently as:

(12a) DELTA(T) (SO) = $[-(ru) + (u\cos\theta) + (v\sin\theta)] / (rs\sin\theta)$

(12b) DELTA(T) (SI) = $[+(ru) - (u\cos\theta) + (v\sin\theta)] / (rs\sin\theta)$

(12c) DELTA(T) (FO) = $[+(u) - (ru\cos\theta) - (rv\sin\theta)] / (rs\sin\theta)$

(12d) DELTA(T) (FI) = $[-(u) + (ru\cos\theta) - (rv\sin\theta)] / (rs\sin\theta)$

To obtain the change (a gain or loss) in minimum separation, as a result of a parallel offset, substitute (12a), (12b), (12c), and/or (12d) into (9), if T and T' have the same sign [if not, revert to (7) and (8), using the identity T' = T + DELTA(T)].

F.6 TREATMENT OF LONGITUDINAL AND LATERAL UNCERTAINTIES

Up to now, we have been assuming the aircraft follow their trajectories exactly. Now, suppose that (once GS has committed to a particular maneuver FO, FI, SO, SI) the aircraft accumulate (by the time they reach the intersection) uncertainties of up to:

±	Lonf	(nmi)	(Longitudinal uncertainty of faster aircraft)
±	Lons	(nmi)	(Longitudinal uncertainty of slower aircraft)
±	Latf	(nmi)	(Lateral uncertainty of faster aircraft)
±	Lats	(nmi)	(Lateral uncertainty of slower aircraft)

Further, assume that all uncertainties add up in the "worst" possible way, to minimize separation.

In this section, the values for DELTA(T) given in (12a) - (12d) for the zero-uncertainty case will be modified to take these uncertainties into account.

First, note that these types of uncertainties affect neither the aircraft's speeds (s or rs), nor their encounter angle θ (see discussion preceding (8)); once again only T, T', and DELTA(T) in (7), (8), (9), (10), (11), and (12) is affected.

The effect of Lonf is that at most [Lonf / (s)] is added to or subtracted from DELTA(T) in (12a) - (12d), because the faster aircraft's arrival at the intersection may be earlier or later (by the time it takes to travel distance Lonf). The effect is independent of whether the maneuver is SO, SI, FO, or FI.

The effect of Lons is that at most [Lons / (rs)] is added to or subtracted from DELTA (T) in (12a) - (12d), because the slower aircraft's arrival at the intersection may be earlier or later (by the time it takes to travel distance Lons). The effect is independent of whether the maneuver is SO, SI, FO, or FI.

The effect of Latf is that:

- B (the slower) reaches the intersection earlier/later by [Latf / (rs $\sin\theta$)]
- A (the faster) reaches the intersection earlier/later by [Latf / (s $\tan\theta$)]

Hence, DELTA(T) changes by at most

± [Latf / (rs $\sin\theta$) - Latf / (s $\tan\theta$)];

the effect is independent of whether the maneuver is SO, SI, FO, or FI.

The effect of Lats is that:

- B (the slower) reaches the intersection earlier/later by [Lats / (rs tanθ)]
- A (the faster) reaches the intersection earlier/later by [Lats / (s sinθ)]

Hence, DELTA(T) changes by at most

$$\pm [\text{Lats} / (rs \tan\theta) - \text{Lats} / (s \sin\theta)];$$

again, the effect is independent of whether the maneuver is SO, SI, FO, or FI.

The four changes to DELTA(T) are independent; the maximum total change is thus given (in (13a)) by adding the four individual changes to DELTA(T). It is convenient to express (13a) using the same denominator as (12a) - (12d):

$$\text{(13a)} \quad \frac{\pm(r\,\text{Lonf}\sin\theta) \pm (\text{Lons}\sin\theta) \pm (\text{Latf}\,(1 - (r\cos\theta))) \pm (\text{Lats}\,(\cos\theta - r))}{(rs\sin\theta)}$$

The total (positive or negative) change in DELTA(T) is maximized when all four terms have the same sign. Since Lonf, Lons, Latf, Lats, r, sinθ, and [1 - (r cosθ)] are all non-negative, and the only possible negative component of (13a) is (cosθ - r), the largest absolute value of (13a) must be:

$$\text{(13b)} \quad \frac{+(r\,\text{Lonf}\sin\theta) + (\text{Lons}\sin\theta) + (\text{Latf}\,(1 - (r\cos\theta))) + (\text{ABS}(\text{Lats}\,(\cos\theta - r)))}{(rs\sin\theta)}$$

Equation (13b) is somewhat pessimistic for small values of θ. In a 15-degree crossing, for instance, one aircraft is unlikely to get a headwind while the other gets a tailwind (as implicitly assumed in (13b)). If the aircraft experience a common wind uncertainty vector, it can be shown that the net induced uncertainty is maximized when the vector is oriented perpendicular to the bisector of the encounter angle. The magnitude of the uncertainty is reduced by a factor of sin(θ/2). This topic is explored in more detail in Appendix G. One models a fraction f ($0 \le f \le 1$) of the longitudinal uncertainty as being aircraft-independent, and the remaining fraction (1 - f) as being due to a common wind error vector. (13b) becomes:

$$\text{(13c)} \quad \frac{(r\,\text{Lonf} + \text{Lons})\sin\theta\,(f + (1 - f)\sin(\theta/2)) + (\text{Latf}\,(1 - r\cos\theta)) + \text{ABS}\,(\text{Lats}\,(\cos\theta - r))}{(rs\sin\theta)}$$

When f = 1, (13c) reduces to (13b).

To determine the effectiveness of a given GS maneuver (SO, SI, FO, FI), the appropriate value of DELTA(T) is first computed using (12), for the zero-uncertainty case. If that value is positive, (13b) or (13c) must be subtracted; is negative, (13b) or (13c) is added. If the error term (13b) or (13c) exceeds (12) in absolute value, the GS maneuver is completely ineffective, since it is unable even to overcome the uncertainty (much less provide safe separation).

Modifying (8) to reflect uncertainty effects (recall T' = T + DELTA(T)), one obtains an expression for D', the minimum separation as a result of a GS maneuver:

$$(14) \quad D = \frac{rs \sin\theta \, [\mathrm{ABS}(T + (12a, b, c, \text{or } d)) - (13b \text{ or } c)]}{\mathrm{SQRT}\,(r^2 - 2r\cos\theta + 1)}$$

In (14), if D' ≤ 0, the maneuver is ineffective assuming adverse uncertainties. The choice of which version of (12) to use is governed by which GS maneuver is under consideration; the choice of which version of (13) to use is governed by how the uncertainty is modeled.

F.7 DETERMINING WHICH GS MANEUVER (SO, SI, FO, FI) IS BEST

Sections F.7 through F.9 present a derivation GS_SEP, the separation achievable via GS, as a function of r, θ, uHI, vLO, vHI, and the uncertainty parameters. The analysis assumes the gentleness of the maneuver is bounded (u ≤ uHI, vLO ≤ v ≤ vHI, as discussed in Section 2.4) and seeks to maximize the separation achievable. The reader who is more interested in determining the gentlest possible GS maneuver to generate a given amount of separation may skip immediately to Section F.10.

One can increase separation between the two aircraft either by

- Getting the FASTER aircraft ahead [alter T via a POSITIVE DELTA(T)]
- Getting the SLOWER aircraft ahead [alter T via a NEGATIVE DELTA(T)]

These two strategies will be labeled the GFA and GSA strategies, respectively (for "Get Faster/Slower Ahead"). In this section, it is determined which of GFA or GSA is "best", in the sense of providing maximum separation subject to the constraint that u ≤ uHI and vLO ≤ v ≤ vHI. For other possible interpretations of "best", see Section F.10.

F.7.1 To Get Slower Ahead (GSA), Best GS Maneuver Is FI (FASTER INSIDE)

The maneuver for which (12i) is most negative (where i = a. b. c. or d) is the best maneuver for GSA. (This is evident from (14), noting that the decrements in separation due to the uncertainty terms, either (13b) or (13c), is independent of u and v, and is independent of which of the four GS maneuvers is chosen.)

One would expect intuitively that FI would be the best GS maneuver to get the slower aircraft ahead of the faster (or further ahead of the faster), because FI simultaneously:

- Delays the faster aircraft, and
- Causes the intersection to occur sooner, thereby reducing the time that the faster aircraft has to catch up with the slower (if the faster is ahead early, the GSA strategy is probably ineffective anyway, even using FI)

In fact, it can be proven mathematically that (12d) (representing FI) is always equal to or more negative than (12a), (12b), or (12c) (representing SO, SI, and FO), for any values of $0 < \theta < 180$, $0 < r \leq 1$, $u > 0$, $v > 0$, and $s > 0$) (note that the denominators of (12a) - (12d) are identical and always positive, since $\sin\theta > 0$ for $0 < \theta < 180$; only the numerators are compared here):

FI vs SO:

(12d) ≤ (12a),	since $-(u) + (ru\cos\theta)$	$\leq -(ru) + (u\cos\theta)$,
	since $-(1) + (r\cos\theta)$	$\leq -(r) + (\cos\theta)$
	since $(-1 - \cos\theta)$	$\leq (-1 - \cos\theta)\, r$

FI vs SI:

(12d) ≤ (12b),	since $-(u) + (ru\cos\theta)$	$\leq +(ru) - (u\cos\theta)$,
	since $-(1) + (r\cos\theta)$	$\leq +(r) - (\cos\theta)$,
	since $(-1 + r)$	$\leq (-1 + r)\cos\theta$

FI vs FO:

(12d) ≤ (12c),	since $-(u) + (ru\cos\theta)$	$\leq (u) - (ru\cos\theta)$,
	since $-(1) + (r\cos\theta)$	$\leq (1) - (r\cos\theta)$,
	since the left side is non-positive and right side non-negative	

In the proofs, the v term is ignored, since it only makes (12d) even more negative with respect to (12a) and (12b), and unchanged with respect to (12c). The effect of uncertainties (13b) - (13c) is ignored, since it is the same for all four GS maneuvers.

Note that the larger u and v are, the better FI is for GSA, since (12d) decreases with increasing values of u and v. To maximize the separation achieved via FI, set u = uHI and v = vHI.

F.7.2 Determining Which of SO, SI, FO to Use To Get Faster Ahead (GFA)

In some geometries, getting the faster aircraft ahead (GFA) is preferable to GSA. For GFA, DELTA (T) should be as large as possible. The maneuver for which (12i) is most positive (i = a, b, c, or d) is the best maneuver for GFA. Of course, FI is never best for GFA, since FI gives the most negative DELTA(T) of the four, as shown in the previous section. This section addresses the question of which of the other three GS maneuvers is best for GFA, as a function of θ, r, uHI, vLO, vHI, and s.

Of course, (12) is expressed using u and v, rather than uHI, vHI, etc. Consider, for example, (12c), for FO. The larger u is, the better FO is for GFA, since (12c) includes the term [u (1 - r cosθ)], and (1 - r cosθ) is always positive. However, the smaller v is, the better FO [illegible] FA, since (12c) includes the term -(r sinθ v), and (r sinθ) is always positive. Hence, to maximize separation for FO, set u = uHI and v = vLO in (12c).

F.7.2.1 SO vs SI

SO is better than SI for GFA whenever (12a) is more positive than (12b), which occurs whenever

$$-(ru) + (u \cos\theta) > +(ru) - (u \cos\theta),$$

which is true if and only if:

$$(15a) \qquad (SO > SI) \qquad \cos\theta > r$$

Note that the larger v is, the better SO and SI are for GFA, since (12a) and (12b) increase with increasing v. Thus, for SO and SI, set v = vHI to maximize separation.

An examination of (12a) and (12b) reveals, however, that a large value for u helps SO only if cosθ > r, and helps SI only if cosθ < r. Fortunately (in the sense of simplifying the derivation), (15a) asserts that SO is better than SI for GFA under exactly that condition under which SO benefits from as large a value of u as possible. The same is true if "SI" and"SO" (and "<" and ">") are reversed. Thus, the only times when a GS maneuver for GFA benefits from not having u as large as possible are exactly the times when that maneuver is inferior for GFA, anyway. Hence, to achieve maximum separation with a gentle maneuver as defined in Section 2.4, u should always be as large as possible, for whatever maneuver is best for GFA (and, as seen in Section F.6.1, for GSA as well).

Thus, separation is maximized by setting u = uHI, for all four GS maneuvers.

F.7.2.2 FO vs SO

FO is better than SO for GFA whenever (12c) is more positive than (12a), which occurs if and only if:

$$u - (ru\cos\theta) - (rv\sin\theta) > -(ru) + (u\cos\theta) + (v\sin\theta)$$

FO benefits from as small a v as possible, while SO benefits from as large a v as possible. For the former, set v = vLO; for the latter, set v = vHI. So, FO is better than SO for GFA if and only if:

$$uHI - (r\,uHI\cos\theta) - (r\,vLO\sin\theta) > -(r\,uHI) + (uHI\cos\theta) + (vHI\sin\theta), \text{ or}$$

(15b) (FO > SO) $uHI\,(1 + r)\,(1 - \cos\theta) > (vHI + r\,vLO)\sin\theta$

F.7.2.3 FO vs SI

FO is better than SI for GFA whenever (12c) is more positive than (12b), which occurs if and only if:

$$u - (ru\cos\theta) - (rv\sin\theta) > +(ru) - (u\cos\theta) + (v\sin\theta)$$

As before, FO benefits from as small a v as possible, while SI benefits from as large a v as possible. For the former, set v = vLO; for the latter, set v = vHI. So FO is better than SI for GFA if and only if:

$$uHI - (r\,uHI\cos\theta) - (r\,vLO\sin\theta) > +(r\,uHI) - (uHI\cos\theta) + (vHI\sin\theta), \text{ or}$$

(15c) (FO > SI) $uHI\,(1 - r)\,(1 + \cos\theta) > (vHI + r\,vLO)\sin\theta$

F.7.2.4 Synthesis: Determining Which of SO, SI, FO To Pick for GFA

If inequalities (15b) and (15c) are both true, then (12c) is the most positive among (12a) - (12d), so that FO is best for GFA. Otherwise, if (15a) is true, then (12a) is the most positive among (12a) - (12d), so that SO is best for GFA. But if (15a) is false, then (12b) is the most positive among (12a) - (12d), so that SI is best for GFA. In short:

```
IF (15b) and (15c) THEN FO
ELSE IF (15a) THEN SO
     ELSE SI
     ENDIF
ENDIF
```

F.8 DECIDING WHETHER TO GET FASTER AIRCRAFT AHEAD (GFA) OR TO GET SLOWER AIRCRAFT AHEAD (GSA)

Let BEST_T_GSA represent (12d), the most negative DELTA(T) available to GS. Let BEST_T_GFA represent max ((12a), (12b), (12c)), the most positive DELTA(T) available to GS (which may be determined via the rules of Section F.7.2.4).

It is now possible to determine, as a function of θ, r, uHI, vLO, vHI, s, and T, whether GSA or GFA provides the most ultimate separation. (T, recall, is the time the slower aircraft reaches the intersection minus the time the faster aircraft reaches it.)

From (8), the final separation D' due to a GS maneuver is proportional to:

ABS(T') = ABS[T + DELTA(T)] = ABS [T + (BEST_T_GFA or BEST_T_GSA)]

Note that no other terms but T' in (8) are affected by the nature of the GS maneuver. This absolute value is maximized via:

- GFA, whenever (T + BEST_T_GFA) > -(T + BEST_T_GSA).
- GSA, otherwise.

Equivalently, GFA provides more separation than GSA if and only if:

(16a) T > - (BEST_T_GFA + BEST_T_GSA)/2.

GSA and GFA provide equally good minimum guaranteed separation for the value of T, denoted T_BREAKEVEN, which renders (16a) an equality:

(16b) T_BREAKEVEN = -(BEST_T_GFA + BEST_T_GSA)/2

It is important to note that T = T_BREAKEVEN is the most pessimistic possible value of T for GS (in the sense that it minimizes D'):

- For larger T, GFA causes (T'), ABS(T'), and hence D' to increase in (8)
- For smaller T, GSA causes (-T'), ABS(T'). and hence D' to increase in (8)

The minimal value of ABS(t') can be calculated as follows:

min [ABS(T')]
= min [ABS (T + DELTA(T))]
= min [ABS(T_BREAKEVEN + (BEST_T_GFA .. or .. BEST_T_GSA)]
= min [ABS (- (BEST_T_GFA + BEST_T_GSA)/2 + (BEST_T_GFA .. or. .BEST_T_GSA))]
= min [ABS ($\pm$ (BEST_T_GFA - BEST_T_GSA)/2]
= (BEST_T_GFA - BEST_T_GSA)/2

Thus:

(17) min [ABS(T')] = (BEST_T_GFA - BEST_T_GSA)/2

It is useful to express (16b) and (17) explicitly in terms of the basic variables θ, r, uHI, vLO, vHI, s and T. One must treat separately the three separate cases for evaluating BEST_T_GFA (which were discussed in Section F.7.2). As discussed in that section, u is set to uHI for all four GS maneuvers, and v is set to vHI for SO, SI, and FI, but to vLO for FO, in order to maximize D' while keeping the maneuvers gentle. After the appropriate version of (12) is substituted for BEST_T_GFA and BEST_T_GSA, one obtains the following:

GFA is a better strategy than GSA (i.e., (16b) is true) if and only if:

(18a) (SO) $T < [uHI\,(1 + r)\ (1 - \cos\theta) - (1 - r)\sin\theta\,(vHI)]\ /\ (2r\,s\,\sin\theta)$

(18b) (SI) $T < [uHI\,(1 - r)\ (1 + \cos\theta) - (1 - r)\sin\theta\,(vHI)]\ /\ (2r\,s\,\sin\theta)$

(18c) (FO) $T < [\ + (r)\sin\theta\,(vHI + vLO)]\ /\ (2r\,s\,\sin\theta)$

(Use (18a), (18b), (18c), respectively, if SO, SI, or FI is best for GSA.)

Note that the uncertainty term from (13b) or (13c) (which is added to each of (12a) - (12d) to account for the most pessimistic possible lateral and longitudinal uncertainties) cancels, since the most pessimistic value for (13b) or (13c) is positive for BEST_T_GSA, and the same value negated for BEST_T_GFA.

F.9 DETERMINING MINIMUM GUARANTEED SEPARATION ACHIEVED BY GS

The next step in the analysis is to derive an expression for GS_SEP, the minimum guaranteed separation achieved by GS as a function of the encounter geometry and the gentleness parameters.

After replacing BEST_T_GFA and BEST_T_GSA with the appropriate version of (12), Equation (17) may be expressed as a function of θ, r, uHI, vLO, vHI, and s:

(19a) min $ABS(T') = [\ uHI\ (1 - r)(1 + \cos\theta) + (1 + r)\ \sin\theta\,(vHI) - 2X]\ /\ (2\,r\,s\,\sin\theta)$

(19b) min $ABS(T') = [\ uHI\ (1 + r)(1 - \cos\theta) + (1 + r)\ \sin\theta\,(vHI) - 2X]\ /\ (2\,r\,s\,\sin\theta)$

(19c) min $ABS(T') + [2uHI\ (1 - r\cos\theta) + (r)\ \sin\theta\,(vHI - vLO) - 2X]\ /\ (2\,r\,s\,\sin\theta)$

(Use (19a), (19b), (19c), respectively, if SO, SI, or FI is best for GSA.)

The variable X is the numerator of (13b) or (13c), the amount by which the uncertainty terms affect GS_SEP.

Using (13b):

- $X = (r\ Lonf\ \sin\theta) + (Lons\ \sin\theta) + (Latf\ (1 - (r \cos\theta))) + (ABS(Lats\ (\cos\theta - r)))$

Using (13c):

- $X = (r\ Lonf + Lons) \sin\theta\ (f + (1 - f)\ \sin(\theta/2)) + (Latf\ (1 - (r \cos\theta))) + (ABS\ (Lats\ (\cos\theta - r)))$

GS_SEP, the lower bound on guaranteed separation achieved by GS, is obtained in (20i) by substituting (19i) into (8), for i = a, b, c. Note that the factor ($r\ s \sin\theta$) nicely cancels.

If SO, SI, or FO is best for GFA, then GS_SEP is respectively (20a), (20b), or (20c):

$$\text{(20a)}\quad GS\_SEP = [\ uHI\ (1 - r)(1 + \cos\theta) + (1 + r)\ \sin\theta\ (vHI) - 2X] / (2Y)$$

$$\text{(20b)}\quad GS\_SEP = [\ uHI\ (1 + r)(1 - \cos\theta) + (1 + r)\ \sin\theta\ (vHI) - 2X] / (2Y)$$

$$\text{(20c)}\quad GS\_SEP = [2\ uHI\ (1 - r \cos\theta) + (r)\ \sin\theta\ (vHI - vLO) - 2X] / (2Y)$$

where Y is the denominator in (8), or $SQRT(r^2 - 2r \cos\theta + 1)$.

F.10 DETERMINING THE GENTLEST GS MANEUVER TO ACHIEVE A GIVEN SEPARATION

Up until now, the goal has been to express GS_SEP as a function of the encounter geometry (r, θ), the uncertainty parameters, and the gentleness parameters (uHI, vHI, vLO). However, it is equally possible to use the above mathematical derivations to go in the reverse direction—that is, to fix the desired separation distance at, say, NEEDED_SEP, and to solve for the maneuver parameters, u and v.

The values of u and v would no longer be bounded by uHI, vHI, and vLO; they would simply be whatever is necessary to achieve NEEDED_SEP. For the most pessimistic geometries (those with T = T_BREAKEVEN), u and v would be gentler than uHI and vHI whenever GS_SEP > NEEDED_SEP, and less gentle whenever GS_SEP < NEEDED_SEP. However, by making u and v functions of T as well, it is possible to take advantage of more favorable values of T, even in geometries where GS_SEP < NEEDED_SEP, and to guarantee NEEDED_SEP via gentle values of u and v.

One could use, as above, (18a) - (18c) to determine which maneuver to use (SO, SI, FO, FI). Then, one would use (14) to solve for u and v.

Alternatively, one could solve for u and v for all four GS maneuvers, and select the best maneuver according to some more sophisticated criterion than "maximize separation". (For example, if FO

provides 5.8 nmi and FI provides 5.7 nmi, but FI causes the aircraft to diverge much earlier than does FO, FI, may be judged superior.)

Substituting (12) and (13) into (14), one obtains:

(21a) (SO) NEEDED_SEP $= [\mathrm{ABS}(-(ru) + (u \cos\theta) + (v \sin\theta) + rsT \sin\theta) - X] / Y$

(21b) (SI) NEEDED_SEP $= [\mathrm{ABS}(+(ru) - (u \cos\theta) + (v \sin\theta) + rsT \sin\theta) - X] / Y$

(21c) (FO) NEEDED_SEP $= [\mathrm{ABS}(+(u) - (ru \cos\theta) - (rv \sin\theta) + rsT \sin\theta) - X] / Y$

(21d) (FI) NEEDED_SEP $= [\mathrm{ABS}(-(u) + (ru \cos\theta) - (rv \sin\theta) + rsT \sin\theta) - X] / Y$

where X is the numerator of (13b) and (13c), and Y is the *denominator of* (14), *or* SQRT $(r^2 - 2r \cos\theta + 1)$.

Any GS maneuver whose u and v satisfy the appropriate version of (21) will provide NEEDED_SEP separation.

If NEEDED_SEP is smaller than $[(\mathrm{ABS}(rsT \sin\theta) - X) / Y]$, then $u = v = 0$ works; no GS maneuver is needed (see Section 2.1.)

Incidentally, if u is set to 0, and (21a) - (21d) is solved for v, one can determine the amount of pure delay needed to solve the conflict (achieved, say, by a vector out from nominal and back). Of course, with $u = 0$, there is no difference here between SO and SI, or between FO and FI.

Note that there is one equation for two variables (u and v), so there are many solutions. One would prefer solutions which are as gentle as possible and realistic for ATC. If one desires to keep the ratio u/v constant, an equation such as:

(22a) $v = 0.2\, u$

can be used as a second equation to determine a particular u and v. Alternatively, if it is desired to keep the distance HJ (here denoted h) constant (and one is willing, for simplicity, to assume the aircraft travels straight from H to K_0 in Figure F-2), the second equation could be $u^2 + h^2 = (v+h)^2$, which is equivalent to:

(22b) $u = \mathrm{SQRT}(2hv + v^2)$

There are other possibilities as well to relate u and v.

F.11 TIMING OF THE GS MANEUVER

As discussed in Section 2.2, the GS analysis assumes that closest separation as a result of a GS maneuver occurs after the full lateral offset u is attained (points K_i and K_o in Figure F-2); GS's mathematical tractability hinges upon this assumption. It is important, however, to assure that the maneuver need not start unreasonably early in order to satisfy this assumption.

The time TAU is defined as the latest time that the maneuvered aircraft can reach its lateral offset u such that:

- The two aircraft are separated by at least GS_SEP at all times equal or prior to TAU.

GS_SEP, as defined above, is the minimum guaranteed separation AFTER time TAU. Hence, if the offset is achieved by no later than time TAU, the aircraft never come within GS_SEP at any time, before, during, or after the maneuver.

TAU is not derived explicitly. Instead, to keep the analysis simple, two lower bounds are derived for TAU, called TAU_x and TAU_y (in Section F.11.1 for FO and FI, and in Section F.11.2 for SO and SI). A summary is given in Section F.11.3.

Assume that the coordinate axes are rotated so that the maneuvered aircraft's original trajectory falls on the y-axis, with the original intersection at the origin. Without loss of generality (thanks to the use of the notation "inside and outside" rather than "left and right", so that θ need only range between 0 and 180 degrees) the trajectory of the unmaneuvered aircraft can be assumed to be oriented in a west (negative x) to east (positive x) direction.

TAU_x and TAU_y are defined as the latest times that the maneuvered aircraft can reach its lateral offset u such that:

- the two aircraft, as projected along the x or y axes, are separated by at least GS_SEP at all times equal or prior to TAU_x or TAU_y, respectively.

Then, $TAU \geq TAU_x$ and $TAU \geq TAU_y$, if the aircraft achieves its offset no later than $\max(TAU_x, TAU_y) \leq TAU$, they never come within GS_SEP at any time before, during, or after the maneuver.

F.11.1 TAU_x and TAU_y for Faster Aircraft's Maneuvers (FO and FI)

For FO and FI, the faster aircraft moves north along the y axis at s nmi/min, reaches the origin at time 0; the slower moves (north/south) west to (south/north) east at speed rs and reaches the origin at time T (Figure F-3a). The next few equations refer to the two aircraft's original (unmaneuvered) trajectories.

The faster aircraft's x- and y-coordinates at time 0, denoted $F_x(0)$ and $F_y(0)$, are:

(23a-1) $F_x(0) = 0$ nmi
(23a-2) $F_y(0) = 0$ nmi

The slower aircraft's coordinates at time 0, denoted $S_x(0)$ and $S_y(0)$, are:

(23b-1) $S_x(0) = -rsT \sin\theta$ nmi
(23b-2) $S_y(0) = -rsT \cos\theta$ nmi

The faster aircraft's x- and y-coordinates increase at rates of:

(23c-1) $d(F_x)/dt = 0$ nmi/min
(23c-2) $d(F_y)/dt = s$ nmi/min

while the slower aircraft's x- and y-coordinates increase at respective rates of:

(23d-1) $d(S_x)/dt = rs \sin\theta$ nmi/min
(23d-2) $d(S_y)/dt = rs \cos\theta$ nmi/min

so that the differences between the slower's and faster's x-coordinates and y-coordinates increase at respective rates of:

(23e-1) $d(S_x\text{-}F_x)/dt = (rs \sin\theta)$ nmi/min (always positive)
(23e-2) $d(S_y\text{-}F_y)/dt = (rs \cos\theta - s)$ nmi/min (always negative)

At any given time t, therefore, the difference between the slower aircraft's and the faster aircraft's x-coordinates and y-coordinates, respectively is:

(23f-1) $$S_x(t) - F_x(t) = S_x(0) - F_x(0) + t\,[d(S_x\text{-}F_x)/dt]\text{nmi}$$
$$= -rsT \sin\theta - 0 + t\,(r\,s \sin\theta)\text{ nmi}$$

(23f-2) $$S_y(t) - F_y(t) = S_y(0) - F_y(0) + t\,[d(S_y\text{-}F_y)/dt]\text{nmi}$$
$$= -rsT \cos\theta - 0 + t\,(r\,s \cos\theta - s)\text{ nmi}$$

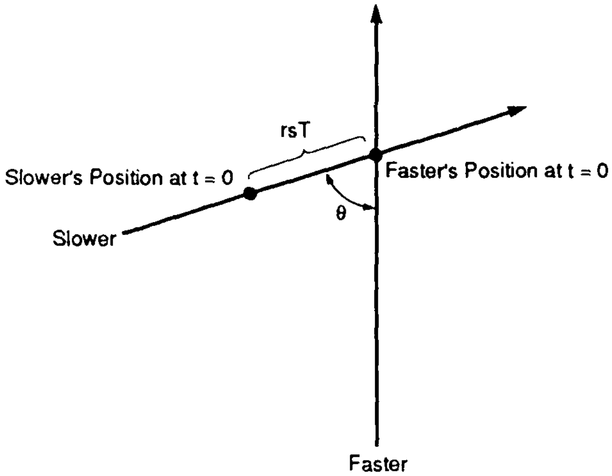

FIGURE F-3a
DETERMINING HOW LATE FASTER AIRCRAFT'S MANEUVER MAY BEGIN

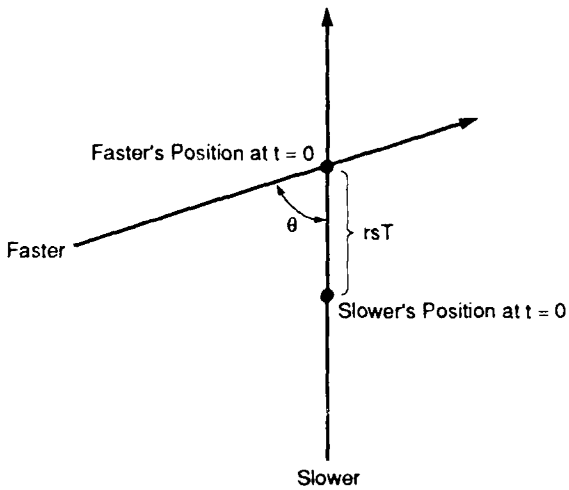

FIGURE F-3b
DETERMINING HOW LATE SLOWER AIRCRAFT'S MANEUVER MAY BEGIN

Equations (23f) can be modified to include the effects of a GS maneuver for the faster aircraft:

$$(23g\text{-}1) \quad S_x(t) - F_x'(t) = -rsT \sin\theta + t\,(rs \sin\theta) \quad - M_x(t)$$
$$(23g\text{-}2) \quad S_y(t) - F_y'(t) = -rsT \cos\theta + t\,(rs \cos\theta - s) \quad - M_y(t)$$

where $F_x'(t)$ and $F_y'(t)$ are the x and y coordinates of the faster aircraft along its maneuver, and $M_x(t) = F_x'(t) - F_x(t)$ and $M_y(t) = F_y'(t) - F_y(t)$.

Note that:

- $M_x(t) = M_y(t) = 0$, if the maneuver is not yet begun at time t;
- $M_x(t) = uk$ nmi, and $M_y(t) = -v$ nmi, if the maneuver is complete by time t, where $k = -1$ for FI and $+1$ for FO.
- If the maneuver is partially complete by time t, $M_x(t)$ lies between 0 and uk, and $M_y(t)$ lies between 0 and -v.

The idea is to find a value of TAU_x and TAU_y (by which time the GS offset is achieved), which is small (early) enough to assure that the aircraft's x- or y-projections ((23g-1) or (23g-2)) prior to that time never come within distance GS_SEP.

However, the values for $M_x(t)$ and $M_y(t)$ are not known (they are functions of how the maneuver is achieved, which depends on the particular aircraft involved). Therefore, it is convenient to insist that the x- or y-projections must be separated by distance GS_SEP for the entire range of possible values for $M_x(t)$ and $M_y(t)$. One value in that range will be the most pessimistic; it will be a minimum or maximum of the entire range. The most pessimistic value for $M_x(t)$ and $M_y(t)$ is respectively denoted M_x and M_y. Thus $M_x = 0$ or uk, and $M_y = 0$ or -v; which value applies depends upon which GS maneuver is being used. For now, M_x and M_y are simply substituted for $M_x(t)$ and $M_y(t)$ in (23g) to yield:

$$(23h\text{-}1) \quad S_x(t) - F_x''(t) = -rsT \sin\theta \quad + t\,(rs \sin\theta) \quad - M_x$$
$$(23h\text{-}2) \quad S_y(t) - F_y''(t) = -rsT \cos\theta \quad + t\,(rs \cos\theta - s) - M_y$$

where $F_x''(t)$ and $F_y''(t)$ represent the maneuvered or unmaneuvered coordinates of the faster aircraft, whichever is more pessimistic in the above sense.

As t becomes more and more negative (representing ever-earlier times to achieve the offset u prior to the conflict), (23h-1) falls and (23h-2) rises. Therefore:

- TAU_x, by definition, is the least negative value of t for which (23h-1) is equal to -GS_SEP. For $t < TAU_x$, (23h-1) will always be less than -GS_SEP. The aircraft therefore cannot approach within GS_SEP prior to TAU_x, because their x-projections do not do so.

- TAU_y, by definition, is the least negative value of t for which (23h-2) is equal to +GS_SEP. For $t < TAU_y$, (23h-2) will always be greater than +GS_SEP. The aircraft therefore cannot approach within GS_SEP prior to TAU_y because their y-projections do not do so.

Hence (substituting t = TAU_x and TAU_y in (23h)),

(23i-1) $S_x(TAU_x) - F_x''(TAU_x) = -GS\_SEP$
$= -rsT \sin\theta + TAU_x\ (r\,s \sin\theta)\ nmi \quad - M_x$

(23i-2) $S_y(TAU_y) - F_y''(TAU_y) = +GS\_SEP$
$= -rsT \cos\theta + TAUy\ (r\,s \cos\theta - s)\ nmi \quad - M_y$

Solving for TAU_x and TAU_y yields (using denominators that are always positive):

(23j-1) $TAU_x = [-GS\_SEP + rsT \sin\theta + M_x] / (rs \sin\theta)$
(23j-2) $TAU_y = [-GS\_SEP - rsT \cos\theta - M_y] / (s\,(1 - r \cos\theta))$

Since TAU_x and TAU_y are lower bounds on TAU, the most "pessimistic" values should be selected for M_x and M_y—that is, values that make TAU_x and TAU_y as negative as possible. M_x should thus be as negative as possible; M_y should be as positive as possible. M_x, which ranges from 0 to uk (k=-1 for FI and k=1 for FO), is therefore set to -u for FI and to 0 for FO. M_y, which ranges between 0 and -v, is therefore set to 0. Thus:

(23k-1) (FI) $TAU_x = (-GS\_SEP + rsT \sin\theta - u) / (rs \sin\theta)$
(FO) $TAU_x = (-GS\_SEP + rsT \sin\theta) / (rs \sin\theta)$

(23k-2) (FO & FI) $TAU_y = (-GS\_SEP - rsT \cos\theta) / (s(1 - r \cos\theta))$

F.11.2 TAU_x and TAU_y For Slower Aircraft's Maneuvers (SO and SI)

For SO and SI, the slower aircraft moves north along the y axis at rs nmi/min, reaches the origin at time T; the faster moves (north/south)west to (south/north)east at speed s and reaches the origin at time 0 (Figure F-3b). The next few equations refer to the two aircraft's original (unmaneuvered) trajectories.

At time 0, the slower aircraft's x- and y coordinates are:

(24a-1) $S_x(0) = 0$ nmi
(24a-2) $S_y(0) = -rsT$ nmi

The faster aircraft's x- and y-coordinates at time 0 are:

(24b-1)	$F_x(0)$	=	0	nmi
(24b-2)	$F_y(0)$	=	0	nmi

The slower aircraft's x- and y-coordinates increase (before and after, but not during, the maneuver) at respective rates of:

(24c-1)	$d(S_x)/dt$	=	0	nmi/min
(24c-2)	$d(S_y)/dt$	=	rs	nmi/min

while the faster aircraft's coordinates increase at respective rates of:

(24d-1)	$d(F_x)/dt$	=	$s \sin\theta$	nmi/min
(24d-2)	$d(F_y)/dt$	=	$s \cos\theta$	nmi/min

so that the differences between the faster's and slower's x-coordinates and y-coordinates increase at respective rates of:

(24e-1)	$d(F_x-S_x)/dt$	=	$(s \sin\theta)$	nmi/min	(always positive)
(24e-2)	$d(F_y-S_y)/dt$	=	$(s \cos\theta - rs)$	nmi/min	(positive if and only if $\cos\theta > r$)

At any given time t, therefore, the difference between the faster aircraft's and the slower aircraft's x-coordinates and y-coordinates, respectively, is:

$$\text{(24f-1)} \quad F_x(t) - S_x(t) = F_x(0) - S_x(0) + t\,(d(F_x-S_x)/dt) \text{ nmi}$$
$$= 0 - 0 + t\,(s \sin\theta) \text{ nmi}$$

$$\text{(24f-2)} \quad F_y(t) - S_y(t) = F_y(0) - S_y(0) + t\,(d(F_y-S_y)/dt) \text{ nmi}$$
$$= 0 + rsT + t\,(s \cos\theta - rs) \text{ nmi}$$

Equations (24f) can be modified to include the effects of a GS maneuver for the slower aircraft:

$$\text{(24g-1)} \quad F_x(t) - S_x'(t) = t\,(s \sin\theta) - M_x(t)$$
$$\text{(24g-2)} \quad F_y(t) - S_y'(t) = +rsT + t\,(s \cos\theta - rs) - M_y(t)$$

where $S_x'(t)$ and $S_y'(t)$ are the x and y coordinates of the slower aircraft along its maneuver, and $M_x(t) = S_x'(t) - S_x(t)$ and $M_y(t) = S_y'(t) - S_y(t)$.

Note that:

- $M_x(t) = M_y(t) = 0$ if the maneuver is not yet begun at time t;

- $M_x(t) = uk$ nmi, and $M_y(t) = -v$ nmi, if the maneuver is complete by time t, where $k = -1$ for SI and $+1$ for SO.

- If the maneuver is partially complete by time t, $M_x(t)$ lies between 0 and uk, and $M_y(t)$ lies between 0 and -v.

The idea is to find a value of TAU_x and TAU_y (by which time the GS offset is achieved), which is small (early) enough to assure that the aircraft's x- or y-projections ((24g-1) or (24g-2)) prior to that time never come within distance GS_SEP.

However, the values for $M_x(t)$ and $M_y(t)$ are not known (they are functions of how the maneuver is achieved, which depends on the particular aircraft involved). Therefore, it is convenient to insist that the x- or y-projections must be separated by distance GS_SEP for the entire range of possible values for $M_x(t)$ and $M_y(t)$. One value in that range will be the most pessimistic; it will be a minimum or maximum of the entire range. The most pessimistic value for $M_x(t)$ and $M_y(t)$ is respectively denoted M_x and M_y. Thus $M_x = 0$ or uk, and $M_y = 0$ or -v; which value applies depends upon which GS maneuver is being used. For now, M_x and M_y are simply substituted for $M_x(t)$ and $M_y(t)$ in (24g) to yield:

$$\text{(24h-1)} \quad F_x(t) - S_x''(t) = -t(s \sin\theta) - M_x$$
$$\text{(24h-2)} \quad F_y(t) - S_y''(t) = +rsT + t(s\cos\theta - rs) - M_y$$

where $S_x''(t)$ and $S_y''(t)$ represent the maneuvered or unmaneuvered coordinates of the slower aircraft, whichever is more pessimistic in the above sense.

As t becomes more and more negative (representing ever-earlier times to achieve the offset u prior to the conflict), (24h-1) falls. (24h-2) falls if $\cos\theta > r$, and rises if $r > \cos\theta$. Therefore:

- TAU_x, by definition, is the least negative value of t for which (24h-1) is equal to -GS_SEP. For $t < TAU_x$, (24h-1) will always be less than -GS_SEP. The aircraft therefore cannot approach within GS_SEP prior to TAU_x, because their x-projections do not do so.

- TAU_y, by definition, is the least negative value of t for which (24h-2) is equal to -GS_SEP (for $\cos\theta > r$) or equal to +GS_SEP (for $r > \cos\theta$). For $t < TAU_y$, (24h-2) will always be less than -GS_SEP (for $\cos\theta > r$) or greater than +GS_SEP (for $r > \cos\theta$). The aircraft therefore cannot approach within GS_SEP prior to TAU_y, because their y-projections do not do so.

Hence (substituting TAU_x and TAU_y for t in (24h)),

(24i-1) $F_x(TAU_x) - S_x''(TAU_x) = -GS\_SEP$
$= + TAU_x \quad (s \sin\theta) - M_x$

(24i-2) $F_y(TAU_y) - S_y''(TAU_y) = -GS\_SEP$ (if $\cos\theta > r$) or $+GS\_SEP$ (if $r > \cos\theta$)
$= + rsT + TAU_y \quad (s \cos\theta - rs) - M_y$

Solving for TAU_x and TAU_y (using denominators that are always positive) yields:

(24j-1) $TAU_x = [-GS\_SEP + M_x] / (s \sin\theta)$

(24j-2) (if $\cos\theta > r$) $TAU_y = [-GS\_SEP + rsT + M_y] / [s (\cos\theta - r)]$
(if $r > \cos\theta$) $TAU_y = [-GS\_SEP - rsT - M_y] / [s (r - \cos\theta)]$

Since TAU_x and TAU_y are lower bounds on TAU, the most "pessimistic" values should be selected for M_x and M_y—that is, values that make TAU_x and TAU_y as negative as possible. M_x should be as negative as possible, as should M_y if $\cos\theta > r$, but M_y should be as positive as possible if $r > \cos\theta$. M_x, which ranges between 0 and uk (k=-1 for SI and k=1 for SO), is therefore set to -u for SI and 0 for SO. M_y, which ranges between 0 and - v, is therefore set to -v for $\cos\theta > r$ and to 0 for $r > \cos\theta$. Thus:

(24k-1) (SI) $TAU_x = (-GS\_SEP - u) / (s \sin\theta)$
(SO) $TAU_x = (-GS\_SEP) / (s \sin\theta)$

(24k-2) (SI & SO; $r > \cos\theta$) $TAU_y = (-GS\_SEP + rsT) / [s (r - \cos\theta)]$
(SI & SO; $\cos\theta > r$) $TAU_y = (-GS\_SEP - rsT - v) / [s (\cos\theta - r)]$

From Equation (15a), SO is used by GS in preference to SI if and only if $\cos\theta > r$. Hence the first part of (24k-2) applies for SO, and the second for SI. Hence:

(24k-3) (SI) $TAU_y = (-GS\_SEP + rsT) / [s (r - \cos\theta)]$
(SO) $TAU_y = (-GS\_SEP - rsT - v) / [s (\cos\theta - r)]$

F.11.3 Timing Issues: Summary

In Sections F.11.1 and F.11.2, the following values have been derived for TAU_x and TAU_y (the sequence is made SO, SI, FO, FI, to match Equation (12), etc.):

$$(25a)\quad (SO)\quad TAU_x = (-GS\_SEP) \;/\; (s \sin\theta)$$
$$(25b)\quad (SI)\quad TAU_x = (-GS\_SEP - u) \;/\; (s \sin\theta)$$
$$(25c)\quad (FO)\quad TAU_x = (-GS\_SEP + rsT \sin\theta) \;/\; (rs \sin\theta)$$
$$(25d)\quad (FI)\quad TAU_x = (-GS\_SEP + rsT \sin\theta - u) \;/\; (rs \sin\theta)$$

$$(26a)\quad (SO)\quad TAU_y = (-GS\_SEP - rsT - v) \;/\; (s (\cos\theta - r))$$
$$(26b)\quad (SI)\quad TAU_y = (-GS\_SEP + rsT) \;/\; (s (r - \cos\theta))$$
$$(26c)\quad (FO)\quad TAU_y = (-GS\_SEP - rsT \cos\theta) \;/\; (s (1 - r \cos\theta))$$
$$(26d)\quad (FI)\quad TAU_y = (-GS\_SEP - rsT \cos\theta) \;/\; (s (1 - r \cos\theta))$$

The TAU_x and TAU_y for the GSA and GSF maneuvers are not, in general, equal. Let:

$$(27a)\quad TAU\_GSA = \max(TAU_x \text{ for GSA maneuver}, TAU_y \text{ for GSA maneuver})$$
$$(27b)\quad TAU\_GFA = \max(TAU_x \text{ for GFA maneuver}, TAU_y \text{ for GFA maneuver})$$

Clearly, GS's decision whether to select GSA or GFA must be made by the earlier of these two times. Further delaying the choice, in effect, amounts to a commitment to the maneuver with the later TAU, since the maneuver with the early TAU may no longer work. Hence, the effective value of TAU is:

$$(28)\quad EFFECTIVE\_TAU = \min(TAU\_GSA, TAU\_GFA)$$

F.12 INTERVAL BETWEEN TIME GS COMMITS AND TIME OF FIRST CLOSE APPROACH

For certain ATC purposes, a value of more interest than $\max(TAU_x, TAU_y)$ itself is:

$$(29)\quad PRE\_TIME = MANEUVER\_TIME + (CLOSE\_TIME - EFFECTIVE\_TAU)$$

PRE_TIME measures the interval elapsed between the time GS must commit to a solution and the time that the aircraft first come close. Selected values of PRE_TIME, as functions of r and θ, are given in Table F-3. PRE_TIME corresponds to the bounds on HJ and JQ (discussed at the end of Appendix E; also see Section 2.10). The two terms of (29) respectively corresponds to the bounds on HJ and JQ. Values of u and v are as shown in Figure 3-3. The discontinuities in PRE_TIME (apparent in Table F-3) occur at boundaries between regions in (r, θ)-space shown in Figures 3-2 and 3-3.

TABLE F-3
PRE_TIME: MINUTES BEFORE CLOSE APPROACH A GS MANEUVER MUST BEGIN

		Speed Ratio (r)				
		.6	.7	.8	.9	1.0
Encounter	20	3.6	2.5	2.6	3.5	4.9
Angle	40	3.4	3.3	3.2	6.9	5.5
Theta	60	3.5	3.5	3.5	4.5	3.8
	80	3.6	3.6	5.1	4.4	3.9
	100	2.5	4.0	3.5	3.0	2.7
	120	4.8	4.0	3.5	3.1	2.7
	140	4.8	4.1	3.5	3.1	2.8
	160	4.8	4.1	3.5	3.1	2.8

PRE_TIME is of more interest than max(TAU_x, TAU_y) for judging whether the GS maneuver must start "unreasonably" early, while TAU_x and TAU_y are more useful in determining exactly how late a specific maneuver may safely start.

MANEUVER_TIME is the length of time it takes to achieve offset u (for u=12 nmi and v=2 nmi, MANEUVER_TIME is about 4 minutes). As long as u is at least several times larger than v, MANEUVER_TIME is closely approximated (via setting v=HK_o - HJ in Figure F-2) by the following expression:

$$\text{(30)} \quad \text{MANEUVER\_TIME} = \frac{u^2 + v^2}{2v\,(\text{speed})}$$

where speed = (s) for "FO vs. FI" and (rs) for "SO vs. FI" and "SI vs. FI" (in the latter two cases, the slower aircraft might have to achieve offset u, which takes longer than the faster aircraft would take). CLOSE_TIME is the time the (unmaneuvered) aircraft would first "come close" (as discussed in Section 2.10 and Appendix E), where "come close" is defined as coming within distance

CLOSE_SEP. (In numerical data presented in Section 3, CLOSE_SEP is set equal to 12 nmi.) CLOSE_TIME can be calculated from (4) by substituting D(t) = CLOSE_SEP and solving for t:

(31) CLOSE_TIME $=$ $[-b - \mathrm{SQRT}(b^2 - 4ac)] / 2a$, where

$$a = s^2 (r^2 - 2r\cos\theta + 1)$$

$$b = s^2 [2rT(\cos\theta - r)]$$

$$c = s^2 r^2 T^2 - (\mathrm{CLOSE\_SEP})$$

APPENDIX G

MODELING OF LONGITUDINAL AND LATERAL UNCERTAINTY IN GS

This appendix discusses the sensitivity of the separation achieved by GS (GS_SEP) to various types of path-prediction uncertainty. The topic divides naturally into two subtopics:

- Early uncertainty, that is resolved by the time GS must commit to a maneuver (this subtopic is important, since significant benefits of the GS analysis, including the GS-MOM interface, are enhanced by early predictability of GS_SEP
- Uncertainty that accumulates after GS commits itself to a maneuver, as parameterized by:
  - Lons (longitudinal uncertainty of the slower aircraft)
  - Lonf (longitudinal uncertainty of the faster aircraft)
  - Lats (lateral uncertainty of the slower aircraft)
  - Latf (lateral uncertainty of the faster aircraft)

G.1 UNCERTAINTY PRIOR TO A GS MANEUVER

Figure G-1 shows the trajectories of two aircraft:

- As planned (perhaps 30 minutes ahead of time)—thick lines
- As might actually be observed (on radar) as time passes—thin lines

G.1.1 Uncertainty in Lateral and Longitudinal Offset

The points H1' and H2' in Figure G-1 represent the predicted positions of the aircraft at the time GS would commit to a maneuver. By that time, however, the aircraft are each typically offset by some distance both longitudinally and laterally, so that they reach points H1 and H2 rather than H1' and H2'. The separation (GS_SEP) achievable by GS is unaffected by such positional displacements, however, they affect neither the encounter angle θ nor the relative speed (from which GS_SEP is calculated).

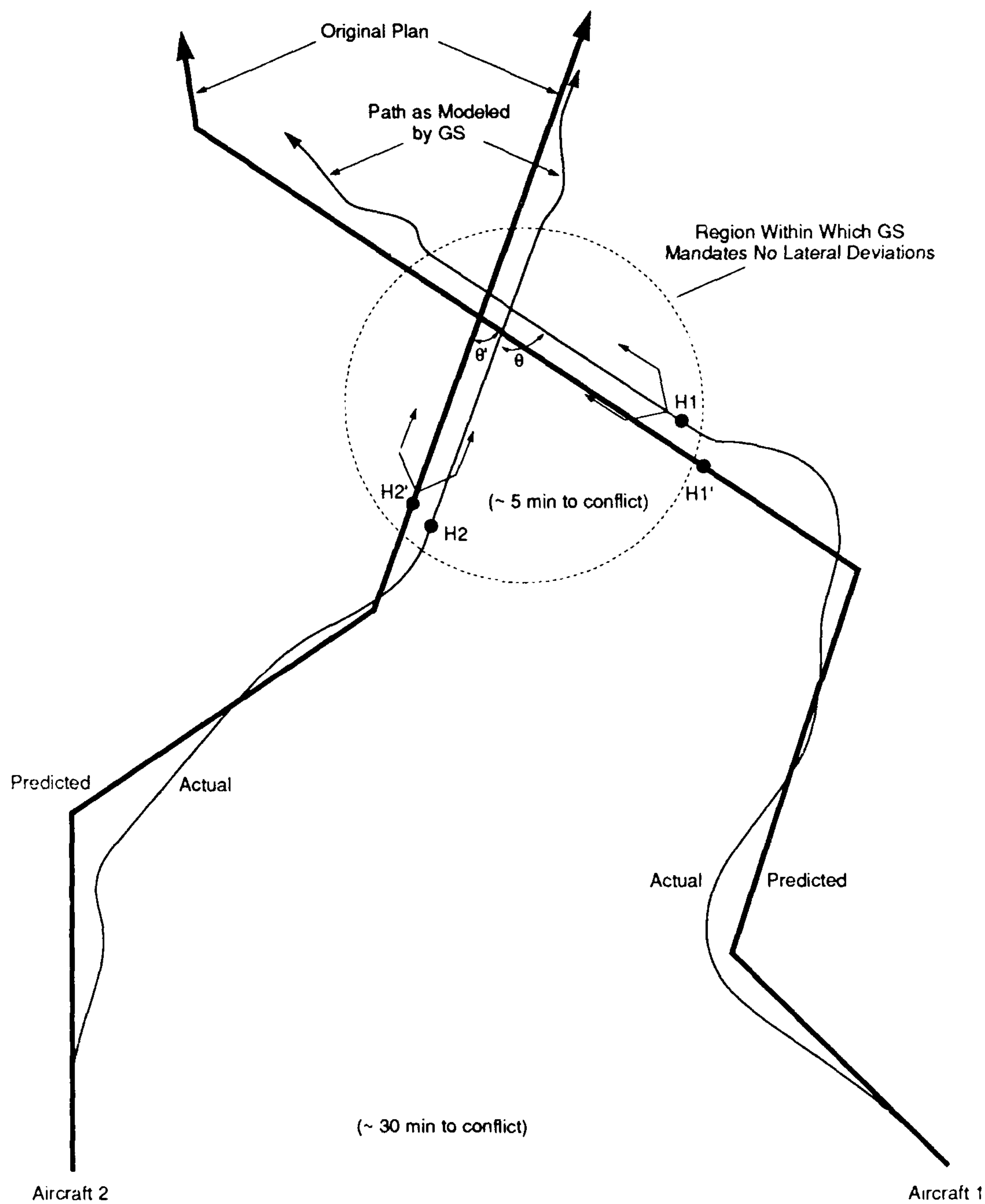

FIGURE G-1
LATERAL AND LONGITUDINAL UNCERTAINTIES PRIOR TO TIME GS COMMITS TO MANEUVER (AT POINTS H1, H2)

G.1.2 Uncertainty in Predicted Encounter Angle and Relative Speed

It may happen, though, that the encounter angle θ (formed by the thin lines) is slightly different from the angle θ' (formed by the thick lines), and that the speed ratio r is slightly different than predicted earlier.

For an ASF application of GS, there is no need to get the aircraft back to the original θ and r; GS simply considers the new geometry as the one to resolve.

A MOM application is sensitive to imperfect predictions of θ and r (causing imperfect predictions of GS_SEP). The uncertainties are considerably smaller than uncertainties in relative timing (the variable T as defined in Section 2.3). For instance, a prediction error of 10 knots in one aircraft's speed represents only about 2 or 2.5 percent of the aircraft's ground speed (for typical en route speeds of around 450 knots); such an error for both aircraft, if they combine adversely, could cause r to be wrong by 4 or 5 percent.

In numerical terms, the uncertainties in predicted values of GS_SEP and predicted closest approach for a conflict 30 minutes in the future might be something like this:

- GS_SEP might be 5.5 nmi with an uncertainty of 0.4 nmi
- Predicted closest approach might be 6 nmi with an uncertainty of 5 nmi

G.2 UNCERTAINTY SUBSEQUENT TO A GS MANEUVER

According to the equation for GS_SEP in Section 3.1 (see also discussion in Appendix F, Section 6), every unit increment in the uncertainty parameters Lons, Lonf, Lats, and Latf causes the following respective decrements in GS_SEP (both expressed in units of distance, such as nmi):

Lons: $(\ \sin\theta)\,((1 + \sin(\theta/2)/2))\ /\ \mathrm{SQRT}\ (r^2 - 2r\cos\theta + 1)$

Lonf: $(r\ \sin\theta)\,((1 + \sin(\theta/2)/2))\ /\ \mathrm{SQRT}\ (r^2 - 2r\cos\theta + 1)$

Lats: $(\mathrm{ABS}(\cos\theta - r))\ /\ \mathrm{SQRT}\ (r^2 - 2r\cos\theta + 1)$

Latf: $(1 - (r\cos\theta))\ /\ \mathrm{SQRT}\ (r^2 - 2r\cos\theta + 1)$

The values assume that the aircraft do not experience their longitudinal uncertainties (Lons and Lonf) independently, but rather that half of the uncertainty is modeled as independent, while half is modeled to be due to an unpredicted wind vector affecting both aircraft. The idea is that aircraft in a 20- or 30-degree crossing geometry are likely to share an unpredicted wind, to some degree, since their headings are similar.

G.2.1 Longitudinal Uncertainty (Lons and Lonf)

In presenting numerical data (in the Section 3 figures), a default value of 1.0 is assumed for Lons and Lonf. Each aircraft is assumed to accumulate 1.0 nmi of longitudinal uncertainty, from the time that the GS maneuver begins to the time of closest approach. This amount is what would accrue given (as an example) a 10 knot discrepancy between predicted and actual along route winds, operating over six minutes.

This small an uncertainty is admittedly optimistic, at least in terms of the longitudinal prediction uncertainties that occur today. However, by the AERA 3 time frame (circa 2000), improvements in accuracy of measured winds and temperatures aloft are expected, via improvements in the nationwide wind/temperature model and/or use of local wind/temperature measurements obtained from the aircraft themselves and forwarded to ATC. AERA 3 may well require such improvements (for reasons unrelated to GS).

There are various ways to model longitudinal uncertainty as a function of θ. The method selected here (rather arbitrarily, with a view to simplicity) is to allow some fraction (f) of Lons and Lonf to be independent of θ, while the remaining (1-f) fraction of Lons and Lonf is due to a shared wind uncertainty vector, whose effect upon in-trail aircraft cancels out, but whose effect increases as θ rises. Thus, in-trail aircraft would be modeled to experience a longitudinal uncertainty not of (Lons) and (Lonf), but of (f Lons) and (f lonf). Aircraft that are not in trail (θ>0) would also experience this contribution to uncertainty due to the fraction (f) of Lons and Lonf that is geometry-independent; however, they also experience a nonzero contribution from the remaining (1-f) fraction.

To illustrate the treatment of the remaining fraction (1-f) of Lons and Lonf, consider a northbound versus an eastbound aircraft (θ = 90). Suppose that the wind uncertainty vector is 10 knots over six minutes, or 1 nmi. Of this 1 nmi, only (1-f) nmi is of concern at the moment. The relative effect of the two aircraft is maximized if the wind direction is northwest or southeast. If so, one aircraft gets a headwind of about 7 knots (of which 7 (1-f) knots is of concern at the moment), while the other gets a tailwind of equal magnitude; the net relative uncertainty vector is 14(1-f) knots. Over the same six minutes, this is 1.4(1-f) nmi.

It can be proven that, for all θ, the direction of the wind error vector that maximizes the relative longitudinal effect on the two aircraft is the direction perpendicular to the bisector of the encounter angle θ, and that maximal effect is (1-f) (Lonf + Lons) (sin(θ/2)).

For each aircraft, then, the decrement to GS_SEP per unit increment in Lons and Lonf is:

Lons: $(\ \sin\theta)(f + (1-f)(\sin(\theta/2))) / \mathrm{SQRT}(r^2 - 2r\cos\theta + 1)$

Lonf: $(r\ \sin\theta)(f + (1-f)(\sin(\theta/2))) / \mathrm{SQRT}(r^2 - 2r\cos\theta + 1)$

Note that these reduce to the above expressions if f=0.5,

Tables G-1 through G-4 show the decrement (in nmi) to GS_SEP per unit (nmi) increment in Lons, Lonf, Lats, and Latf, respectively (f is assumed to be 0.5).

G.2.2 Lateral Uncertainty

The lateral uncertainties Lats and Latf are set (to determine the numerical results in Section 3) to 0.5 nmi each. The choice of these values, somewhat smaller than typically assumed today, reflects improved navigation, communication and surveillance. The choice also takes advantage of strict monitoring of aircraft pathkeeping, implemented in AERA 3, upon which GS relies (Section 2.1).

Unpredictability in aircraft lateral positions today has three distinct sources:

Source 1: The pilot tries to hold planned centerline exactly, but cannot (due to limitations in equipment, skill, etc.). A typical magnitude might be 0.5 nmi off centerline.

Source 2: The pilot drifts gradually away from centerline, but does not much care. Often, the controller is not bothered, either. A typical magnitude might be 3 nmi off centerline.

Source 3: The pilot deliberately deviates from the planned centerline, to cut a corner, to avoid weather, etc. A typical magnitude might be 10 nmi off centerline.

Suppose two aircraft would pass safely, provided both avoid rightward lateral deviations from centerline (say, over the next six minutes). Once the pilots are so informed, rightward deviations due to Sources 2 and 3 should be zero (knowing there is traffic on the right, a pilot would avoid weather, etc., via other maneuvers, e.g., a leftward deviation).

Situations in which a directive to "avoid lateral deviations in a particular direction" actually works (though often difficult for today's controllers to determine manually), are readily detected via automation. The automation is therefore likely to take advantage of this method of reducing lateral uncertainties.

Consider next the possible rightward deviations due to the remaining source, Source 1. The automation should be able to reduce these deviations by continually and strictly monitoring each aircraft's adherence to its centerline, and uplinking a message to the pilot should an aircraft go over its line in the unsafe direction. Such strict monitoring on a routine basis would probably be too heavy a workload to impose upon a human controller.

TABLE G-1

DECREMENT TO GS_SEP (nml) PER nml INCREMENT IN LONS

R = Theta	0.500	0.600	0.700	0.800	0.900	1.000
0.1	0.00	0.00	0.00	0.00	0.01	0.45
15	0.27	0.33	0.39	0.48	0.55	0.56
30	0.51	0.56	0.60	0.62	0.63	0.61
45	0.66	0.68	0.69	0.69	0.67	0.64
60	0.75	0.74	0.73	0.71	0.68	0.65
75	0.78	0.76	0.73	0.70	0.67	0.64
90	0.76	0.73	0.70	0.67	0.63	0.60
105	0.71	0.67	0.64	0.60	0.57	0.55
120	0.61	0.58	0.55	0.52	0.49	0.47
135	0.49	0.46	0.43	0.41	0.39	0.37
150	0.34	0.32	0.30	0.28	0.27	0.26
165	0.17	0.16	0.15	0.15	0.14	0.13
179	0.00	0.00	0.00	0.00	0.00	0.00

TABLE G-2

DECREMENT TO GS_SEP (nml) PER nml INCREMENT IN LONF

R = Theta	0.500	0.600	0.700	0.800	0.900	1.000
0.1	0.00	0.00	0.00	0.00	0.01	0.45
15	0.14	0.20	0.28	0.38	0.49	0.56
30	0.25	0.33	0.42	0.50	0.57	0.61
45	0.33	0.41	0.48	0.55	0.60	0.64
60	0.37	0.45	0.51	0.57	0.61	0.65
75	0.39	0.46	0.51	0.56	0.60	0.64
90	0.38	0.44	0.49	0.53	0.57	0.60
105	0.35	0.40	0.45	0.48	0.52	0.55
120	0.31	0.35	0.38	0.41	0.44	0.47
135	0.24	0.27	0.30	0.33	0.35	0.37
150	0.17	0.19	0.21	0.23	0.24	0.26
165	0.09	0.10	0.11	0.12	0.12	0.13
179	0.00	0.00	0.00	0.00	0.00	0.00

TABLE G-3

DECREMENT TO GS_SEP (nml) PER nml INCREMENT IN LATS

R = Theta	0.500	0.600	0.700	0.800	0.900	1.000
0.1	1.00	1.00	1.00	1.00	1.00	0.00
15	0.87	0.82	0.72	0.54	0.25	0.13
30	0.59	0.47	0.32	0.13	0.07	0.26
45	0.28	0.15	0.01	0.13	0.26	0.38
60	0.00	0.11	0.22	0.33	0.42	0.50
75	0.24	0.33	0.42	0.49	0.55	0.61
90	0.45	0.51	0.57	0.62	0.67	0.71
105	0.62	0.66	0.70	0.74	0.77	0.79
120	0.76	0.79	0.81	0.83	0.85	0.87
135	0.86	0.88	0.89	0.91	0.92	0.92
150	0.94	0.95	0.95	0.96	0.96	0.97
165	0.98	0.99	0.99	0.99	0.99	0.99
179	1.00	1.00	1.00	1.00	1.00	1.00

TABLE G-4

DECREMENT TO GS_SEP (nml) PER nml INCREMENT IN LATF

R = Theta	0.500	0.600	0.700	0.800	0.900	1.000
0.1	1.00	1.00	1.00	1.00	1.00	0.00
15	0.97	0.94	0.87	0.74	0.49	0.13
30	0.92	0.85	0.75	0.61	0.44	0.26
45	0.88	0.81	0.71	0.61	0.50	0.38
60	0.87	0.80	0.73	0.65	0.58	0.50
75	0.87	0.82	0.77	0.72	0.66	0.61
90	0.89	0.86	0.82	0.78	0.74	0.71
105	0.92	0.89	0.87	0.84	0.82	0.79
120	0.94	0.93	0.91	0.90	0.88	0.87
135	0.97	0.96	0.95	0.94	0.93	0.92
150	0.99	0.98	0.98	0.97	0.97	0.97
165	1.00	1.00	0.99	0.99	0.99	0.99
179	1.00	1.00	1.00	1.00	1.00	1.00

When analyzing Source 1 uncertainty, in the absence of uncertainty from Sources 2 and 3, the pilot may be assumed to make a bona fide effort to obey the directive to avoid rightward deviations (e.g., there would be no sudden rightward deviations). However, the pilot's (navigation-based) interpretation of maintaining his/her original heading may differ from the ground-based ATC's (surveillance-based) interpretation.

A very small discrepancy (e.g., 5 degrees) might not be detected quickly (due to radar and tracker noise), but once it is, the lateral overshoot and the uplinked correction would be small. A larger discrepancy (e.g., 30 degrees) would be discovered quickly enough that the lateral overshoot would be small. The worse the discrepancy, the more quickly and surely it can be discovered.

Once one lateral correction is issued, the need for a second may be reduced. For example, the pilot may be told to maintain the heading 85 degrees. The pilot sets the autopilot to 85 degrees. However, the track observed by ATC appears to be maintaining a heading of 82 degrees, perhaps due to a 3-degree bias in the pilot's navigation system. The autopilot can be fine-tuned to 88 degrees following the first correction, and ATC will thereafter observe the track to be maintaining 85 degrees as desired.

The conclusion is that AERA 3 should be able to assume minimal uncertainties in the lateral dimension. GS, designed to take full advantage of this fact, relies heavily upon the lateral dimension to achieve separation.

GLOSSARY

AERA	Automated En Route Air Traffic Control, a project sponsored by the Federal Aviation Administration (FAA)
AERA 2	AERA's second phase, in which the controller remains in the loop to resolve short-term problems
AERA 3	AERA's third phase, in which humans contribute to planning decisions but are not involved in resolving short-term problems; GS is an AERA 3 function
ATC	Air Traffic Control
Crossing Conflict	A conflict between two aircraft in which the two trajectories intersect
Conflict	A situation in which two aircraft may need maneuvers to assure safe separation
Complex Problems	Multiple conflicts or multiple possible conflicts
Downstream	Pertaining to air traffic situation more than a few minutes hence
FAA	Federal Aviation Administration
Faster Inside	A GS maneuver whereby the faster aircraft maneuvers toward the inbound portion of the slower aircraft's trajectory
Faster Outside	A GS maneuver whereby the faster aircraft maneuvers away from the inbound portion of the slower aircraft's trajectory
FI	Faster Inside
FO	Faster Outside
Gentle	Describes a resolution maneuver whose displacement from nominal is bounded by parameters
GFA	Get Faster Ahead; the strategy of achieving safe separation by having the faster aircraft reach the intersection first.
GS	Gentle Strict

GSA	Get Slower Ahead; the strategy of achieving safe separation by having the slower aircraft reach the intersection first.
Induced Delay	The along-route delay caused by a parallel offset maneuver
Inside Maneuver	FI or SI
Isolated One-on-One Conflict	A conflict whose resolution is not affected by nearby traffic or other obstacles (storms, etc.)
Latf	Lateral plus/minus uncertainty for the faster aircraft
Lats	Lateral plus/minus uncertainty for the slower aircraft
Lonf	Longitudinal plus/minus uncertainty for the faster aircraft
Lons	Longitudinal plus/minus uncertainty for the slower aircraft
Maneuver Option Manager	A long-lookahead AERA 3 function designed to simplify complex problems
MOM	Maneuver Option Manager
NEEDED_SEP	The desired amount of separation between aircraft
Outside Maneuver	SI or FI
r	The ratio of the slower aircraft's ground speed to that of the faster
s	The faster aircraft's ground speed
SI	Slower Inside
Slower Inside	A GS maneuver whereby the slower aircraft maneuvers toward the inbound portion of the faster aircraft's trajectory
Slower Outside	A GS maneuver whereby the slower aircraft maneuvers away from the inbound portion of the faster aircraft's trajectory
SO	Slower Outside

Strict	Aircraft pathkeeping within precisely-specified bounds
T	The difference between the time the slower aircraft reaches the intersection and the time the faster aircraft does so (no GS maneuver)
THETA	The encounter angle (θ)
Trajectory	The expected path through (x, y, x, t) taken by an aircraft
u	The magnitude of a GS parallel lateral offset maneuver
uHI	The largest parallel lateral offset qualifying as "gentle"
users	Those who benefit from ATC—the public, airlines, etc.
v	Symbol for induced delay (in nmi)
vLO	Lower limit on induced delay, v
vHI	Upper limit on induced delay, v
X	A term expressing the effect upon GS_SEP of longitudinal and lateral uncertainties

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